New Theory of Thermal Stability in Boiling Systems

A quantitative derivation of the criterion for thermal stability provides explicit guides to avoid unstable conditions; it also indicates that the traditional graphical analysis may yield erroneous results

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I have derived a quantitative criterion for thermal stability that affords greater insight into the behavior of reactors and boilers than has previously been available with the old graphical method. The new theory presents a quantitative description of the phenomenon known as burnout and promises to be of direct practical significance in the design of both liquid-cooled reactors and boiling systems in the nuclear field. Among other things it leads to the realization that boiling-heat-transfer coefficients are not very helpful in designing boilers and suggests that the boiling data available in the literature be reanalyzed to correlate the heat flux as a function of the thermal driving force. Within the limits of the space available, I will outline the development of the quantitative theory (detailed proofs are omitted at several points) and demonstrate its application to reactor design with emphasis on once-through liquid-metal boilers—a type of boiler that is becoming prominent in the nuclear space-power program and in which the thermal stability problem is especially crucial.

The practical importance of thermal stability (see box) has been recognized since 1934 when Nukiyama (*1*) first observed the "pool boiling curve" shown in Fig. 1a. Since that time, the theory of thermal stability has centered about the graphical demonstration that, if the curve in Fig. 1a is replotted for a monotonically increasing heat flux, the wall temperature will be discontinuous, as shown in Fig. 1b. This discontinuity in the wall temperature is called "burnout" due to the fact that the wall temperature at point C' often exceeds the melting point of the wall material.

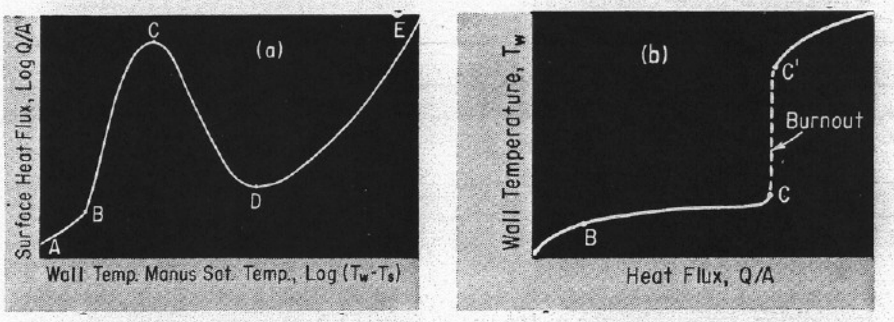


FIG. 1. POOL BOILING CURVE (a) replotted in terms of wall temperature (b) shows discontinuity that in graphical method is assumed to be region of burnout

The above graphical theory does not rigorously apply to real systems and has led to a number of erroneous conclusions about thermal stability. The quantitative theory outlined in this article allows us to see that the following widely accepted conclusions are not always true:

- In reactors and other "fixed heat input" systems, burnout occurs at point C in Fig. 1a.
- Systems in which the temperature of the heat source is controlled necessarily possess thermal stability. In such systems, there is no phenomenon similar to burnout.
- Thermal instability leads to temperature discontinuities but does not lead to oscillatory performance.

These erroneous conclusions have fostered a number of related erroneous conclusions in the fields of reactor design, boiler design, evaporator design, heat transfer to cryogenic and refrigerant fluids, design of heat-transfer test sections and experiments, and correlations of experimental results. Unfortunately, it is not possible to treat each of the above subjects in the space available. Our major emphasis will be on the application of thermal stability to the design and analysis of reactors and reactor plant systems with

Thermal Stability—What Is It?

Heat transfer is a process that involves the flow of heat from a source to a sink. As long as the heat flow leaving the source is just equal, on an instantaneous basis, to the heat flow entering the sink, the system is thermally stable. When this equality is not satisfied, a finite quantity of heat is necessarily stored up in the system or withdrawn from it. This change in heat content can result in either:

- The phenomenon known as burnout in which excess heat builds up to the point where it can cause melting of a reactor fuel element.
- Oscillatory behavior caused by the alternating sign of the heat storage term.

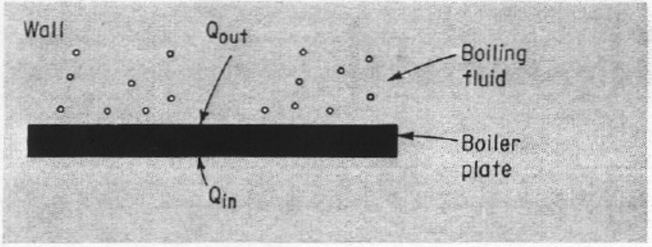


FIG. 2. IDEALIZED thermal system with no net heat flow provides basis for establishing stability criterion of Eq. 9

particular attention to once-through liquid-metal boilers of the kind being considered for use in space.

Definition of Stability

Thermal systems may be classified into two broad categories:

- Systems with no net circulation (such as pool boilers) (Fig. 2)
- Systems with net circulation (such as liquid-metal power plants or forced-convection reactors)

In a practical sense, we are more concerned with the latter group. However, both are thermal systems and because the first type is much simpler, the quantitative derivation will be based on the first type. The results will then be extended to obtain conclusions about the second type of system. In addition, for each of the above systems if the temperature of the heat source is controlled, the system is usually referred to as a "constant temperature" system. If some other attribute of the heat source is controlled, the system is usually referred to as a "fixed heat input" system. Although these conventions will be used throughout this article, it must be emphasized that they are often misleading and are used here only for convenience.

To develop the quantitative theory we will begin by defining thermal stability and then analyze a simple system to determine under what conditions it is not stable. By analyzing a simple system, it is possible to obtain a simple stability criterion and determine the salient features of thermal stability without the cumbersome complication required when treating a complex system. Through an understanding of the salient features of thermal stability, the theory can easily be extended to complex systems. Moreover, this understanding leads to the realization that the designer, through the use of simple techniques, can vastly improve the thermal stability of heat-transfer equipment such as reactors and boilers.

An adequate definition of thermal stability would seem to be the following: A heat source (such as the boiler plate in Fig. 2) is thermally stable at a particular temperature $T(0)$ provided that, when $T(0)$ is perturbed to $T(0) + \Delta T$, the following relationship is satisfied:

$$T(t = \infty) = T(0) \qquad (1)$$

If the result of the perturbation does not satisfy Eq. 1, the heat source is thermally unstable.

The above definition strongly suggests that stability be appraised by perturbing the temperature of the heat source and observing its response.

If the temperature of the boiler plate in Fig. 2 is perturbed from some initial value, it may respond in any of the ways shown in Fig. 3. If the system being analyzed behaves like Curves d, c, and e in Fig. 3 (i.e., dT/dt does not change sign anywhere in the interval $0 < t < \infty$), the stability can be easily appraised by determining the sign of dT/dt. For these idealized cases, the stability criteria become:

- the system is stable if dT/dt is opposite in sign to ΔT
- the system is unstable if dT/dt is of the same sign as ΔT
- the system is unstable if $dT/dt = 0$.

It is unfortunate that real systems do not necessarily behave like Curves d and e. However, instability can be most easily analyzed by idealizing the real system in such a way that it behaves like Curves d and e. After the idealized problem is solved, it becomes an easy matter to remove the idealized aspects and solve the real, physical problem.

It will be noted that the above approach is followed throughout this article. Without this simplification, we would almost certainly become bogged down with the period and amplitude of the oscillations which occur in the interval $0 < t \infty$ and which would not help us appreciate or identify the real problem.

Derivation of Criterion

The above discussion was partly intended to illustrate that thermal stability criteria can be simply stated only for simple systems. Toward this end,

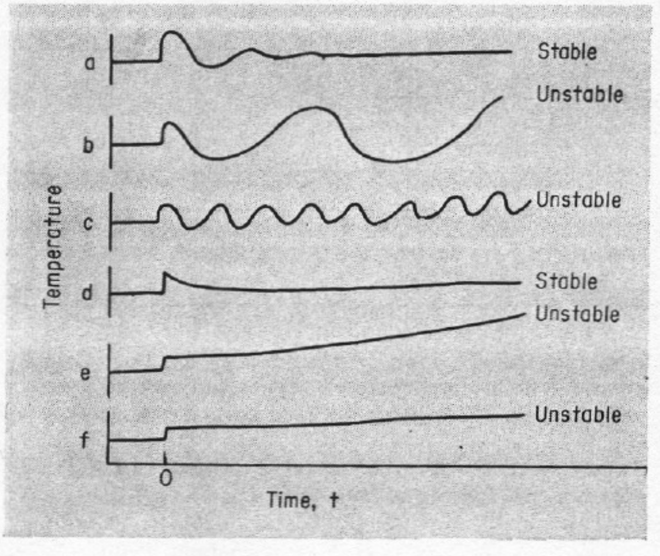
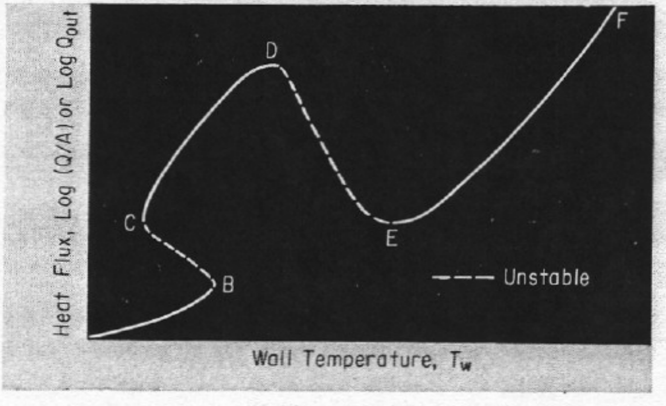


FIG. 3. **MODES OF TRANSIENT RESPONSE** for idealized system shown in Fig. 2. All are unstable except Modes *a* and *d*

FIG. 4. **UNSTABLE REGIONS** of hypothetical pool boiling curve (below) as defined by criterion of Eq. 9 when Eq. 10 is satisfied



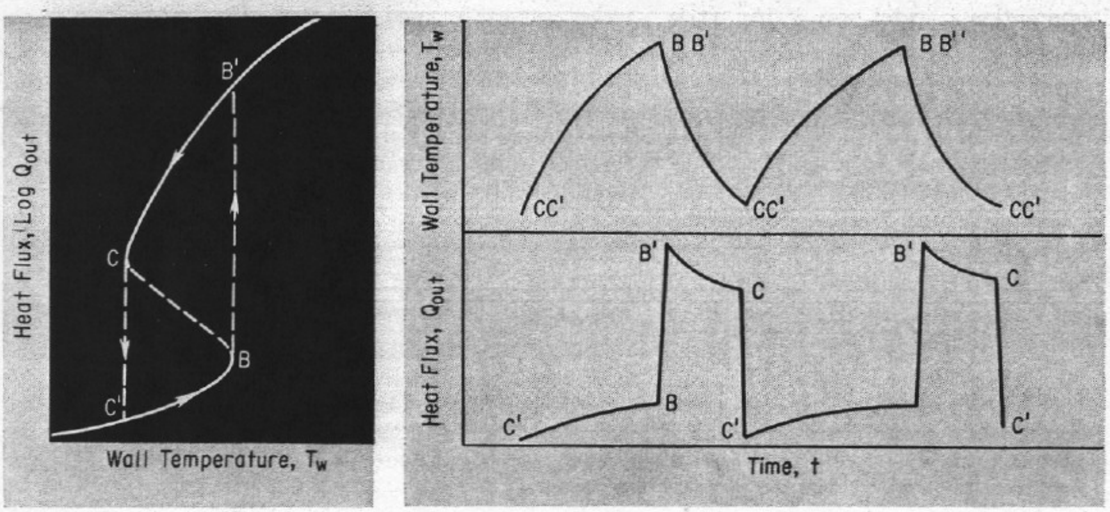


FIG. 5. OSCILLATIONS. Slight perturbation of system operation on curve in Fig. 4 can yield boiling curve at right and resulting system oscillations (far right)

let us now analyze a very simple system which is amenable to hand calculation. However, we must bear in mind that the resultant criterion will be applicable only to systems that closely resemble the idealized system. The system to be analyzed is the pool boiler shown in Fig. 2 and it is idealized by assuming that:

- the temperature of the boiling fluid is constant—i.e., is unaffected by Q_{out}
- the thermal conductivity of the boiler plate is infinite
- the heat capacity of the boiler plate is finite.

In the above idealized system, the parameter we wish to investigate is the temperature of the boiler plate. As suggested by the definition of stability, we shall perturb the temperature and determine under what conditions the temperature is stable. The analysis is based on noting that

$$Q_{in}(t) - Q_{out}(t) = Q_{stored}(t) \quad (2)$$

$$Q_{stored}(t = 0) = 0 \quad (3)$$

When the boiler plate temperature T_w is perturbed a differential amount from some initial value at which Eq. 3 is satisfied,

$$Q_{out}(t) = Q_{out}(\theta) + (dQ_{out}/dT_w)[\Delta T_w(t)] \quad (4)$$

$$Q_{in}(t) = Q_{in}(\theta) + (dQ_{in}/dT_w)[\Delta T_w(t)] \quad (5)$$

Therefore, from Eqs. 2, 3, 4, and 5,

$$Q_{stored}(t) = [\Delta T_w(t)](dQ_{in}/dT_w - dW_{out}/dT_w) \quad (6)$$

Now

$$Q_{stored}(t) = (dT_w/dt)(C_{bp}) \quad (7)$$

where C_{bp} is the total heat capacity of the boiler plate.

From Eqs. 6 and 7

$$dT_w/dt = (1/C_{bp})(dQ_{in}/dT_w - dQ_{out}/dT_w)[\Delta T_w(t)] \quad (8)$$

From Eq. 8, it can be seen that $T_w(0)$ is stable *only* if

$$(dQ_{in}/dT_w - dQ_{out}/dT_w) \underset{s}{<} 0 \quad (9)$$

since this is the only condition under which $T_w(t)$ will tend toward $T_w(0)$, thus satisfying the definition of stability. Equation 9 also demonstrates that the graphical theory is quantitatively correct only in those rare cases in which

$$dQ_{in}/dT_w = 0 \quad (10)$$

Equation 9 is the stability criterion for the idealized system. For the case where Eq. 10 is satisfied, Eq. 9 indicates that the boiler plate temperature would be unstable everywhere in intervals BC and DE of Fig. 4.

Fig. 4 shows that, if the system is initially operating in regionDE, a slight perturbation will cause it to translate to either region CD or EF where it will operate in a stable manner. If the system is initially operating in region BC, a slight perturbation will cause it to translate and it will begin to oscillate (because it cannot "find" a stable condition) as shown in Fig. 5.

Equation 9 applies equally to both "fixed heat input" systems and "constant temperature" systems. The difference in stability between the two types of systems is only one of degree—dQ_{in}/dT_w is generally more negative in constant temperature systems, resulting in greater thermal stability as may be seen from Eq. 9. Thus, burnout does indeed occur in both types of systems and the only significant difference between the two types is that, at burnout the temperature cannot increase without limit in a constant temperature system.

To date, burnout has occurred in constant-temperature pool boilers but seems to have gone unnoticed. However, it has often been observed in forced convection systems, but is usually referred to as a "dry-wall" phenomenon caused by a change in flow regime. Equation 9 demonstrates that this dry-wall phenomenon is simply burnout in a constant-temperature system, and is not caused by a change in flow regime. The fact that a change in flow regime has been visually observed to occur in coincidence with the dry-wall phenomenon would seem to indicate that the heat transfer is affecting the flow regime, rather than the converse.

Applications to Design

Liquid-cooled reactors are normally designed to avoid burnout for obvious reasons. This is accomplished by empirically correlating the results of burnout experiments in which the heat source is electrical rather than nuclear. In the early days of burnout experiments, the burnout point was detected by the physical destruction of the test section. Later, "burnout detectors" were utilized to sense the changing temperature of the heat source and then quickly reduce the power to preserve the test section. The difficulty with both of these methods was that they *assumed* that the burnout heat flux in a reactor would be the same as that in an electrically heated test section. Inspection of Eq. 9 indicates that this assumption is valid only if Eq. 11 is satisfied:

$$(dQ_{\text{in}}/dT_{\text{w}})_{\text{test section}} = (dQ_{\text{in}}/dT_{\text{w}})_{\text{reactor}} \qquad (11)$$

The left hand side of Eq. 11 is primarily a function of the temperature coefficient of electrical resistivity of the test section material, while the right hand side is a function of the temperature coefficient of reactivity of reactor fuel and moderator; hence it seems highly unlikely that Eq. 11 would ever be satisfied.

The latest design concept replaces burnout with DNB (Departure from Nucleate Boiling). DNB occurs before burnout and is detected by observing the decrease in $dQ_{\text{out}}/dT_{\text{w}}$ which occurs prior to burnout. By using this conservative approach in the experiments, the effective result is the same as assuming that the right hand side of Eq. 11 has a large, positive value. In most water cooled reactors, one could expect the right hand side of Eq. 11 to have a negative value, leading to the conclusion that the DNB design concept is indeed conservative. Moreover, by determining the magnitude of $dQ_{\text{in}}/dT_{\text{w}}$ for a given reactor one could appraise the degree of conservatism inherent in all the above methods.

A more important result of the derivation of the thermal stability criterion is the realization that the designer can vastly improve the stability of equipment by the use of simple design techniques. For instance, it was previously thought that with electrical heat input to the boiler plate in Fig. 2 it would be impossible to avoid the temperature discontinuity shown in Fig. 1b. Equation 9 shows that this is not true and that the discontinuity could be avoided altogether by selecting a material with a sufficiently large, positive temperature coefficient of resistivity. The required value would be whatever is necessary to satisfy Eq. 9 at all points of the pool boiling curve. A number of other equally simple techniques can be used to improve the stability of reactors and other types of heat transfer equipment.

Reactor instability. As we have seen above, under certain conditions thermal instability can result in the oscillatory performance of a pool boiler. If these same conditions exist in a reactor, the reactor also can be expected to operate in an oscillatory manner through the following reasoning:

- If the subcooled region of a reactor channel exhibited the behavior shown in region BC of Fig. 4, the heat flux into the coolant would be expected to oscillate as shown in Fig. 5.
- As a result of the oscillations in the heat flux, the enthalpy of the coolant leaving the channel would oscillate.
- If the coolant leaving the channel is only slightly subcooled, the oscillations in enthalpy would cause the void fraction in the channel to oscillate.
- The oscillation in void fraction would cause the hydraulic resistance of the channel to oscillate, thus resulting in an oscillatory channel flow rate. The amplitude of such oscillations might go unnoticed in a forced convection reactor operating at fluid velocities as high as 15–30 ft/sec. However, in a free convection reactor operating at 1–3 ft/sec, the relative amplitude would be much greater and the oscillations easily detected.
- If a sizable fraction of the channels operate near zero subcooling, they would soon begin to oscillate together by virtue of the fact that they are coupled through the reactor kinetics and the reactor flow rate. Moreover, this sizable fraction would cause the reactor power to oscillate in response to the oscillating void fraction in the reactor core. In the same manner, the reactor flow rate might begin to oscillate.

A region such as BC of Fig. 4 would be expected to occur near the inception of boiling, if at all. Now, for water-cooled systems, there is very little evidence in the literature to suggest a region such as BC. However, as pointed out by Bergles and Rohsenow (*2*),

> Investigators frequently neglect to include data for the transition region, or knee of the boiling curve, which is always present between incipient boiling and fully developed boiling.

Moreover, Ref. *2* presents data for a pressurized water system in which the curve of heat flux vs. wall temperature is exactly like region ABCD of Fig. 4. In conjunction with this data (Fig. 8 in Ref. 2), the authors state

> It appears to be no coincidence that the wall temperatures behave strangely when the first bubbles grow.

as predicted by the above theory of thermal stability.

The data of Corty and Foust (*3*) also indicate a region similar to BC of Fig. 4. They found that, in non-aqueous systems, regions similar to BC would occur at the inception of boiling if

> all active (nucleating) centers are thoroughly snuffed out before heat transfer is increased again.

Upon increasing the heat flux from the above condition, they found that

> Superheats of 40–50°F above the saturation temperature of the liquids were possible with no bubbles on the heat-transfer surface even though the normal ΔT for violent nucleate boiling was only about 25°F. Such excessive superheats could be maintained for several minutes, but upon further increases the surface spontaneously broke into vigorous nucleate boiling, and the ΔT then decreased to the "normal" value.

The results of Corty and Foust thus demonstrate the existence of a BC region for non-aqueous pool boilers, but indicate that the region exists only for those cases in which the active centers are initially snuffed out. It seems reasonable to expect that a forced convection system would supply this snuffing action and that region BC would exist in the steady-state, as suggested by the results of Ref. *2*.

In summary, there is a small amount of data that suggests reactor power oscillations may be thermally induced in the manner described by the above theory. Whether this is the major contributor to reactor instability cannot be determined without extensive further analysis. The thermal basis

Comparison of Burnout in Constant-Temperature and Fixed-Heat-Input Systems

	Burnout result
Fixed-heat-input system	Step increase in surface temperature accompanied by change in heat flux
Constant-temperature system	Step decrease in surface heat flux accompanied by increase in surface temperature

for reactor instability is at least suggested by the following excerpt from Ref. *8* by Anderson and Lottes

> Borax IV was different from the other Borax reactors in its fuel assembly material and design. Whereas the first three reactors had used aluminum-uranium alloy fuel elements with a corresponding large thermal conductivity, Borax IV used fuel pellets of mixed uranium oxide and thorium oxide having a low thermal conductivity. Borax IV, with these fuel elements, exhibited a greater degree of stability under similar conditions than did the other Borax reactors. These elements can apparently contribute to reactor stability under certain conditions.

Liquid-metal boilers. Liquid metal boilers (with the exception of pool boilers) are extremely rare. To properly design a once-through liquid-metal boiler to operate at high exit quality, it is essential to gain a thorough understanding of thermal stability. Such a boiler is unique in two aspects, both of which are intimately related to thermal stability

- Their boilers have high exit quality—usually avoided in ordinary boilers.
- Boiling liquid-metal systems exhibit the features of thermal instability to a larger degree than any other class of systems.

The liquid-metal boilers for space will probably be of the constant-temperature type with the heat being supplied by a primary fluid that circulates through a reactor. The high exit quality proposed for the boiler will distinguish it from most common boilers, which are usually designed to operate at low quality followed by a drying stage. By operating in the low quality region, it has been possible to avoid the "dry-wall" phenomenon mentioned briefly above. This phenomenon is the direct result of thermal instability rather than the result of a change in flow regime. Moreover, it is the direct parallel of burnout in fixed heat input systems as shown in the table.

The step decrease in surface heat flux referred to in the table is well demonstrated in the results of Berensen (*4*) presented in Ref. *5*. In Berensen's constant-temperature system, burnout was accompanied by a decrease in surface heat flux from an initial value of 100,000 Btu/hr/ft^2 to values ranging from 4,000 to 15,000 Btu/hr/ft^2 for different surface finishes.

Probably the major reason why the true nature of the dry-wall phenomenon has not been generally recognized to date is the widespread acceptance of correlating heat-transfer coefficients as a function of vapor quality in forced convection systems. With such a correlation, it is extremely difficult to appraise the thermal stability of the system because $dQ_{\text{out}}/dT_{\text{w}}$ cannot be easily determined from a correlation which describes the relationship between heat-transfer coefficient and vapor quality. The use of heat-transfer coefficients does no particular harm if the boiler is being designed to operate in a region remote from thermal instability (such as at low vapor quality). However, in a once-through boiler operating at very high exit quality, the use of heat-transfer coefficients masks the most important effect—thermal stability—and such a boiler should be designed on the basis of heat flux rather than heat-transfer coefficients. Indeed, for similar reasons, all forced convection boiling data should be correlated on the basis of heat flux and thermal driving force, and all boilers should be designed on these bases also.

This problem of burnout in the boiler is one of the major problems confronting the design of the liquid-metal boiler. Heretofore, the problem has simply been circumvented by the use of a drying stage. Indeed, the problem of burnout in a constant temperature boiler has previously had so little practical importance that most experimenters correlate their results only in the region removed from burnout and there seem to be no correlations which apply to the dry-wall phenomenon. (Indeed, it would be virtually impossible to obtain such a correlation by correlating heat-transfer coefficient as a function of quality.) However, several experimenters are investigating the use of swirl devices to improve the thermal stability of the high quality region of the boiler. The results obtained by Gambill et al (*6*) on the use of such devices indicate that they will indeed improve thermal stability.

The conclusion that boiling liquid-metal systems exhibit the requirements for thermal instability to a very high degree is obtained through inductive reasoning as follows:

- Liquid-metal pool boilers have often been observed to operate in an oscillatory manner.
- If such a boiler operates in an oscillatory manner, it seems reasonable to conclude that the oscillations are thermally induced.
- If the oscillations are thermally induced, then it is probably true that $dQ_{\text{out}}/dT_{\text{w}} < 0$.

Thus, it seems reasonable to conclude that the pool boiling curves for liquid metals are often similar to the shape of the curve in Fig. 4. The final verification of this shape must of course await the design and construction of a boiler which is stable in region BC.

A continuation of the above reasoning leads to the conclusion that a forced-convection liquid metal boiler might oscillate in the manner described above for reactors. Such oscillations have indeed been reported by Smith, Tang, and Ross (*7*).

If one applies the concepts of thermal stability to improve the design of once-through liquid metal boilers one is led to the rather surprising conclusion that such boilers should be concurrent heat exchangers rather than the normal countercurrent heat exchangers.

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