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NUCLEATE BOILING--THE RELATIONSHIP BETWEEN HEAT FLUX AND THERMAL DRIVING FORCE

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ABSTRACT

It has been widely accepted that, during nucleate boiling, the heat flux is related to the thermal driving force in a non-linear fashion. This article examines the experimental evidence which led to this result and concludes that

> The nucleate boiling data in the literature do not generally support the contention that the heat flux and the thermal driving force are related in a non-linear fashion. Rather, the data examined indicate that this relationship is essentially linear.

This reexamination indicates that the seeming non-linearity resulted from the vagarious nature of log-log graph paper rather than from the experimental evidence.

INTRODUCTION

The primary purpose of this article is to obtain a definitive answer to the following question:

HOW IS HEAT FLUX RELATED TO THERMAL DRIVING FORCE DURING NUCLEATE BOILING?

This question has of course been posed and answered many times in the past and has almost invariably resulted in the conclusion that these two parameters are related in a non-linear fashion. The relationship most often suggested is that the heat flux is a function of the thermal driving force raised to some power larger than one. A review of the literature indicates that this result is commonly based on plotting the data on log-log graph paper and noting that the resultant slope is oftentimes a value of three or four. As described below, this approach is somewhat lacking in generality because the use of log-log graph paper demands a tacit assumption about the relationship between the two parameters. Moreover, unless the accuracy of this assumption is verified, the resultant slope may be grossly in error.

In the present article, we will take a more general approach to the question. We will analyze the data already available in the literature and will make no assumptions about the relationship between these two parameters. We will attempt to determine this relationship by simply plotting the experimental results on linear graph paper and then using the resultant curves to suggest their own relationships.

A more or less general equation relating two parameters is the type given as equation (1):

$$y = mx^a + b \tag{1}$$

Because of its simplicity, we will first consider an equation of this form where the heat flux is the dependent variable and the thermal driving force is the independent variable as in equation (2):

$$q/A = M(\Delta T)^a + B \tag{2}$$

If the data suggest an equation of this form, we will then attempt to simplify this general equation by determining one or all of the constants. However, we may find that one or all of the constants are affected by parameters such as surface condition or pressure and that equation (2) is already in its most simplified form.

As described below, the analysis results in the realization that the heat flux is essentially linearly related to the thermal driving force during nucleate boiling. This result disagrees not only with the previous data reduction but also disagrees with parts of the present theory of the boiling process. This latter discrepancy will require that certain of the present theory be revised to agree with the experimental evidence.

BACKGROUND

In the past thirty years, a great amount of research and analysis has been performed on the subject of boiling. A recent summary of this large amount of work is presented by Rohsenow in reference (1). With regard to the relationship between heat flux and thermal driving force during nucleate boiling, he states on page 117:

> Although the slope is predominantly in the neighborhood of 3, observations are available with resulting slopes of as low as unity for contaminated surfaces and as high as approximately 25 for very clean surfaces (2).

(In the above, the word "slope" refers to the slope of q/A vs. ΔT plotted on log-log graph paper.) On this same subject, Rohsenow states on page 125:

> The great increase in heat transfer rate associated with nucleate boiling $(q/A) \sim (\Delta T)^3$ is due to the agitation . . .

In a recent discussion of nucleate boiling, Zuber (3) states:

> More than two dozen equations have been heretofore proposed for correlating experimental data . . . . All of them can be put in the form
>
> $$h = \text{const.}\ (\Delta T)^m$$
>
> where the value of the exponent varies between 1 and 3 and the . . . .

(It should be noted that if the above expression had been written for q/A rather than h, the value of the exponent would have been between 2 and 4.) Zuber also notes that

> . . . variations in the exponent m, ranging from 1 to 25 can be produced by polishing the surface with different grades of emery paper . . .

The above summaries indicate that there is fairly widespread agreement that the heat flux is related to the thermal driving force in a non-linear fashion in which the exponent on the thermal driving force varies over a very wide range but is usually some value around three.

In addition to the above empirical correlations, a number of investigators have taken a theoretical approach to determine the relationship between heat flux and thermal driving force. Examples of theoretical treatments are the analyses of Zuber (3) and that of Levy (4). Levy's analysis results in an equation which relates heat flux to the third power of the thermal driving force. Zuber derives an equation relating heat flux to the thermal driving force raised to some power larger than 5/3. (It is not possible to determine the precise value of Zuber's exponent because his expression includes the surface density of nucleating sites as an empirically measured quantity. Since this surface density is positively correlated with the thermal driving force, his derived exponent is always larger than the exponent he derives for the explicit functionality which is 5/3. The fact that part of the functionality between the two parameters is expressed only implicitly of course places a severe limitation on the usefulness of his equation--i.e. it cannot be used unless one first empirically determines the functionality between this surface density and the other parameters in the system.)

In summary, there has been fairly widespread agreement on both an empirical and theoretical basis that, during nucleate boiling, the heat flux is non-linearly related to the thermal driving force. This non-linearity is such that a in equation (2) would be expected to assume values anywhere from 1 to 25 but would generally be between 2 and 4.

DATA ANALYSIS

The data analysis begins by simply plotting up some experimental results on linear graph paper such as is shown in Figures 1 through 3. Once the points are plotted and the shape of the curves has become more or less apparent, we must then determine that relationship which seems most strongly suggested by the data. For the ten "runs" shown in Figures 1 through 3, this is a simple matter because the nucleate boiling portions of the curves strongly suggest that straight lines be drawn through the data. Thus, all ten of the runs suggest that the heat flux is essentially linearly related to the thermal driving force by the relationship

$$q/A = M(\Delta T)^1 + B \qquad (3)$$

The fact that the several runs obviously exhibit various values for M and B suggests that these constants are not pure constants and that they are indeed functions of such variables as surface condition, pressure, and the many other variables encountered in boiling systems.

It should be emphasized that the scatter which is always present in experimental data makes it impossible to prove that the relationship is precisely linear--i.e. we cannot infer that the exponent is 1.000000 simply because the data are well correlated by perfectly straight lines. Rather, the data demonstrates that the exponent is not sufficiently different from unity that the difference can be detected. It therefore seems reasonable to accept the value of 1.0 for the exponent even though any value between 0.9 and 1.1 would probably fit the data equally well.

Close inspection of Figures 1 through 3 reveals that, of the 62 nucleate boiling points plotted, virtually every one falls within 2 F. of the straight lines drawn through the data. Moreover, the average deviation from the straight lines is less than 1 F. This amazing precision demonstrates that any non-linearity in the data is of little practical significance.

It is important to note that the data in Figures 1 through 3 is a fairly widespread sample of nucleate boiling data-- the data consist of data obtained

by two different investigators
in two different laboratories
for two different boiler types
for two different boiler plate materials
for three different fluids
for four different pressures
for five different surface finishes

The fact that every run exhibits a highly linear relationship between the variables in spite of the widely different nature of the runs strongly suggests that our results are generally applicable.

It may perhaps be more convincing to observe that the data in Figures 1 through 3 would give reasonably straight lines if plotted on log-log paper and that the slopes would indeed be in the neighborhood of three or four. For instance, the same data shown in Figure 3 is also presented by Rohsenow on page 119 of reference (1) as a log-log graph in which straight lines are drawn through the nucleate boiling data points. On this particular graph, the slope of the lines varies from about two to six, seeming to suggest that the relationship is non-linear in spite of the fact that it is highly linear as shown in Figure 3. The reason for this seeming anomaly is presented below.

LOG-LOG GRAPH PAPER

It was mentioned above that the use of log-log graph paper involves a tacit assumption about the relationship between the variables of interest. This may be seen by noting that equation (1) will be a straight line on linear graph paper only if a = 1.0 and will be a straight line on log-log graph paper only if b = 0.0. Thus, when we attempt to draw a straight line through data on log-log graph paper, we tacitly assume that b does indeed equal zero. Thus, our result will be quantitatively correct only if we verify the accuracy of this tacit assumption. This of course is easily done by plotting the data on linear graph paper and extrapolating the data to determine the value of b. If b does not equal zero, the data would indicate the wrong value of the exponent if it were plotted directly on log-log graph paper and the slope were measured. This is precisely why the data in Figure 3 indicate an exponent of 1 on linear graph paper and an exponent of 2 to 6 when plotted on log-log paper. As a result of the finite value of B in equation (2), the exponents deduced from the slope on log-log paper are incorrect. (Even a cursory glance at Figures 1 through 3 will indicate that B is indeed finite.)

The above is not to say that one is helpless in the face of a finite value of B. In such a case, the variables simply require a minor transformation so that the transformed equation will exhibit a zero value for B. After transforming the data, the log-log graph would exhibit a slope in agreement with the true value of the exponent. This transformation has been performed on the data in Figure 3 and the results are presented in Figure 4. The transformation has been accomplished by extrapolating the lines in Figure 3 to determine ΔT_o and then subtracting this value from each of the corresponding data points. It should be noted in Figure 4 that the transformed results are satisfactorily correlated by lines drawn with unity slope. Thus, we obtain the same exponent from plotting the data on both linear and log-log graph paper (as desired).

The importance of first performing this transformation is best illustrated by noticing that an exponent of as high as 25 can be deduced from a set of data in which the exponent is actually 1. The safest way to avoid this type of error would seem to be to always plot the data first on linear graph paper.

RESULTS

The obvious result of the above is the realization that heat flux and thermal driving force seem to be linearly related during nucleate boiling and that this result isgenerally applicable. This will simplify the correlation of this type of data and will somewhat simplify the design of boiling equipment (in the sense that it is more convenient to work with linear correlations than with non-linear ones).

A less obvious but far more important result is the fact that this linear relationship is in violent disagreement with some of the present theory about the boiling process. For instance, the separate analyses of Zuber and Levy both resulted in a highly non-linear relationship between heat flux and thermal driving force (as mentioned above). Since this result disagrees with the experimental evidence, it is manifest that the theory on which these analyses were based was incorrect.

It is also interesting to note that Berenson's data is linear with constant slope throughout the entire nucleate boiling region. This strongly suggests that there is only one heat transfer regime of nucleate boiling in this particular data. This therefore tends to disprove the generality of Zuber's (3) theory of bubble interference since this theory resulted in the conclusion that there were generally two heat transfer regimes in nucleate boiling.

THEORETICAL DISCUSSION

Up to this point, the discussion has been completely empirical and has been based solely on the measured data. However, it is appropriate to consider whether there is any other basis which would support or explain our unexpected results and perhaps answer the questions:

1) Why is B in equation (2) finite (i.e. non-zero)?

2) Why is the relationship between heat flux and thermal driving force seemingly linear?

A quite satisfactory answer to the first question can be obtained by first considering the definition of boiling offered by Rohsenow (1):

> The process of evaporation . . . results in the conversion of a liquid into a vapor. When this conversion occurs within a liquid, forming vapor bubbles, it is called boiling.

From this definition it follows that the essence of the process we call boiling is the formation of bubbles within a liquid. Since bubbles in a liquid phase have a natural tendency to collapse due to surface tension, the boiling process must somehow supply a counterforce of finite magnitude in order to form bubbles against the force of surface tension. It is generally agreed that this counterforce is the result of superheat in the liquid. Therefore, since the counterforce must be of finite magnitude, it follows that the superheat required for boiling must also be of finite magnitude. It is this requirement of finite superheat which causes B in equation (2) to be of finite magnitude.

In a very real sense, ΔT_o may be considered as the minimum thermal driving force which can sustain boiling. Thus, if we accept the seemingly inescapable conclusion that the boiling process requires a finite thermal driving force, we are forced to also conclude that B in equation (2) is finite. Once we accept that B in equation (2) is finite, we are forced to conclude that the slope obtained by plotting the data directly on log-log graph paper is not indicative of the relationship between heat flux and thermal driving force.

A more or less satisfactory answer to the second question can be obtained by first observing that the process of nucleate boiling sharply decreases the resistance to heat transfer.

This decrease is generally agreed to be the result of the turbulence created at the boiling interface by the continual formation of bubbles. Therefore we may reasonably conclude that the relationship between heat flux and bubble induced turbulence iswhat in turn determines the relationship between heat flux and thermal driving force. Let us therefore turn our attention to the following parameters which might be expected to amply define this bubble induced turbulence:

a) specific bubble frequency--i.e. number of bubbles per unit time per active nucleating site

b) bubble diameter

c) surface density of nucleating sites

By considering the manner in which each of the above is affected by heat flux, we will attempt to indirectly infer the relationship between heat flux and thermal driving force.

Perkins and Westwater (5) observed that both the bubble frequency and the bubble diameter were essentially independent of average heat flux throughout most of the nucleate boiling region. Thus we must conclude that, once an active site begins to nucleate, further increases in average heat flux have no effect on either the specific bubble frequency or thebubble diameter. This absence of effect makes it necessary to conclude that the relationship between heat flux and bubble induced turbulence is determined solely through the surface density of nucleating sites.

Turning now to the surface density of nucleating sites, Adiutori (6) demonstrates that the data in the literature generally demonstrates a high degree of linearity in the relationship between heat flux and the surface density of nucleating sites. In addition, it seems reasonable to suppose that this surface density is defined by the thermal driving force and that it is only indirectly related to the heat flux. In other words, we suppose that

$$n/A \propto \Delta T^{c} \qquad (4)$$

$$\Delta T \propto (q/A)^{d} \qquad (5)$$

(4) and (5): transformed to eliminate additive constants

and therefore (by substitution)

$$n/A \propto (q/A)^{cd} \qquad (6)$$

From the experimental observation that n/A is linearly related to heat flux, we are forced to conclude that

$$cd = 1 \qquad (7)$$

Equation (7) therefore confronts us with a choice:

a) Either d may assume a wide range of values and the identity is preserved because c always assumes the value 1/d or

b) both c and d are generally equal to essentially unity

Since the first choice seems so untenable, it would seem that the second choice should be accepted by default. Therefore, since we conclude d is generally unity, we have concluded that the heat flux and the thermal driving force should be linearly related. The fact that we have reached this conclusion quite aside from any measurements of thermal driving force tends to support the conclusion and the rationale of the linear relation between heat flux and thermal driving force during nucleate boiling.

CONCLUSIONS

The major conclusions resulting from the above analysis are:

1. The nucleate boiling data in the literature do not generally support the contention that the heat flux and the thermal driving force are related in a non-linear fashion.
2. A reasonably extensive sample of nucleate boiling data taken from the literature suggests that the relationship between heat flux and thermal driving force is essentially linear during nucleate boiling.
3. In general, data should not be plotted on log-log paper unless it is first demonstrated that there is no additive constant such as b in equation (1).
4. Theoretical considerations strongly suggest that nucleate boiling correlations should include an additive constant in order to account for the fact that boiling requires a thermal driving force of finite magnitude.

SYMBOLS

A	area
a,b,c,d	dimensionless constants
B	dimensional constant
h	heat transfer coefficient
m	dimensionless constant
M	dimensional constant
n	number of active nucleating sites

q	heat
T	temperature
ΔT	thermal driving force
ΔT_o	thermal driving force corresponding to zero heat flux (obtained by extrapolating nucleate boiling data)
x,y	unspecified variables

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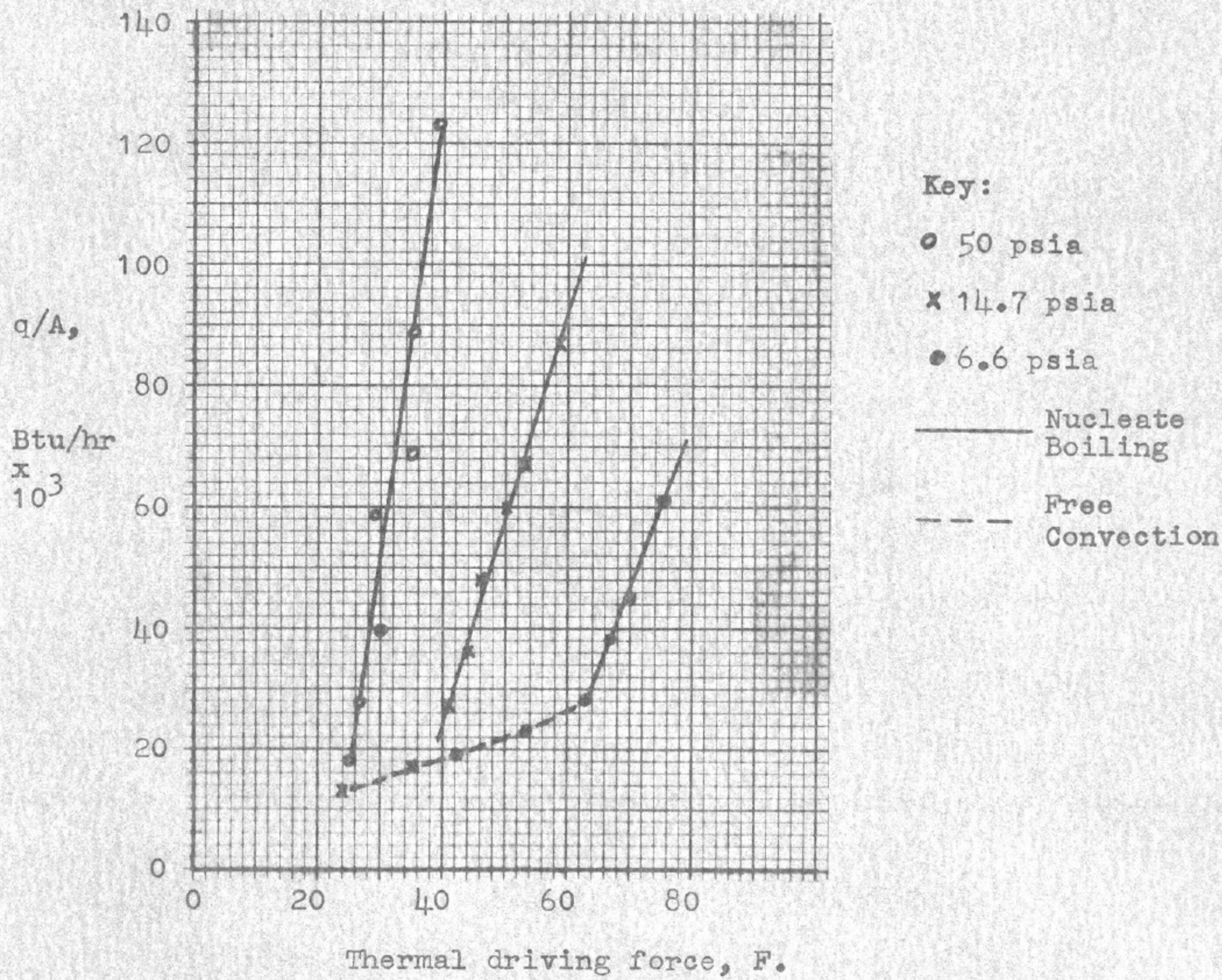

FIGURE 1

Heat Flux vs. Thermal Driving Force, n-Heptane, at Several Pressures (Reference 5 data, Cichelli & Bonilla)

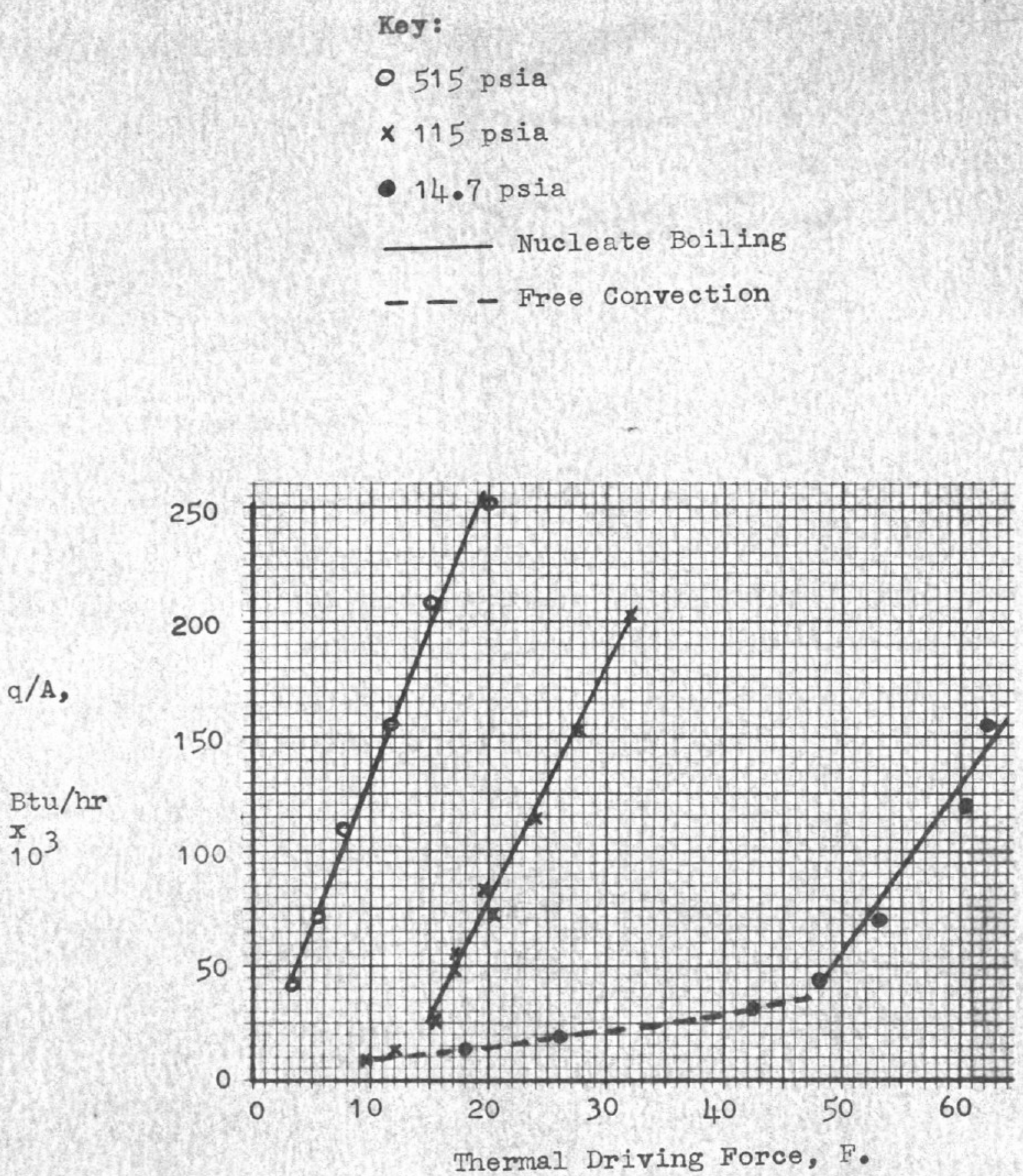

FIGURE 2

Heat Flux vs. Thermal Driving Force, Ethyl Alcohol, at Several Pressures (Reference 5 data, Cichelli & Bonilla)

Key: ~~LAP E~~

● Run ~~31: Emery 320~~ OK 20°

□ Run 32: Emery 60 12°

▲ Runs ~~17 &22: Lap E~~ 31 Emery 300 OK 8°

× Run 2: Mirror Finish 24°

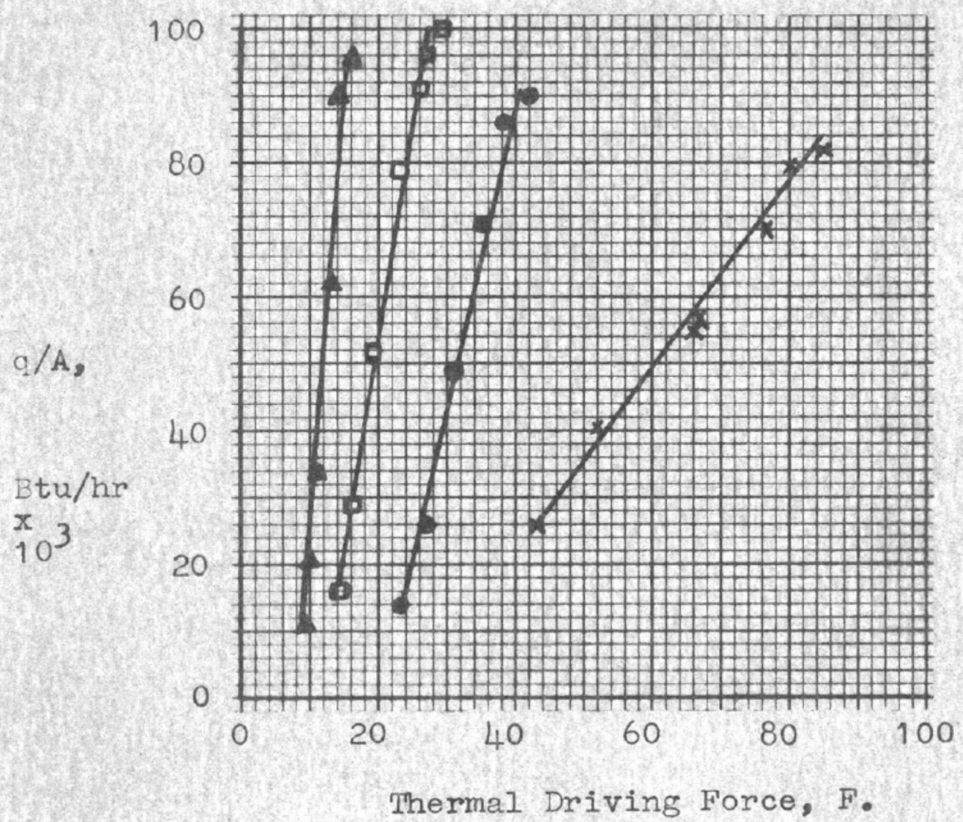

FIGURE 3

Heat Flux vs. Thermal Driving Force, n-pentane, for various Surface Finishes (Reference 6 data, Berenson)

Key: same as Figure 3

Note: All lines drawn with unity slope

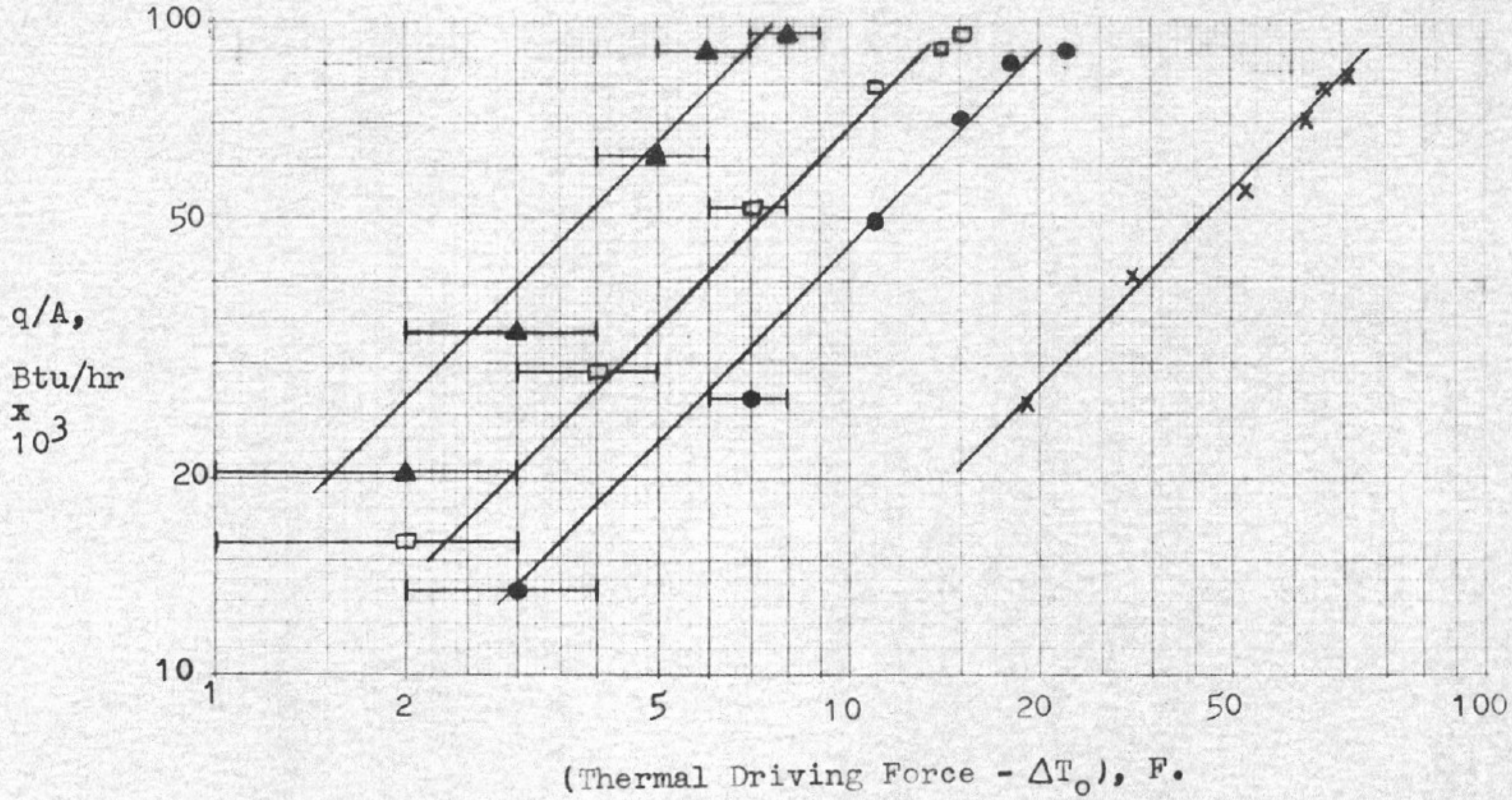

FIGURE 4

Correlating the Data of Figure 3 by First Transforming the Variables