March 13, 1964

NUCLEATE BOILING--THE RELATIONSHIP BETWEEN HEAT FLUX AND THERMAL DRIVING FORCE

by

Eugene F. Adiutori, President
Stability Consultants
Box 18062
Cincinnati 18, Ohio

ABSTRACT

It has been widely accepted that, during nucleate boiling, the heat flux is related to the thermal driving force in a non-linear fashion. This article examines the experimental evidence and the analysis which lead to this result. This reexamination leads to the rather surprising conclusion that

> During nucleate boiling, the heat flux is LINEARLY related to the thermal driving force.

It indicates that the seeming non-linearity resulted from the vagaries of log-log graph paper rather than from the experimental evidence. This new understanding of the relationship between these parameters creates a number of discrepancies between theory and measurement which will have to be resolved by revising the presently accepted theories about the boiling process.

INTRODUCTION

The primary purpose of this article is to obtain a definitive answer to the following question:

How is heat flux related to thermal driving force during nucleate boiling?

This question has of course been posed and answered many times in the past and has almost invariably resulted in the conclusion that these two parameters are related in a non-linear fashion. The relationship most often suggested is that the heat flux is a function of the thermal driving force to some power larger than one. A review of the literature indicates that this result is commonly obtained by plotting the experimental results on log-log graph paper and noting that the resultant slope is oftentimes a value of three or four. As described below, this approach is somewhat lacking in generality because the use of log-log graph paper demands a tacit assumption about the relationship between the two parameters. Moreover, unless the accuracy of this assumption is verified, the resultant slope may be grossly in error.

In the present article, we will take a more general approach to the question. We will analyze the data already available in the literature and will make no assumptions about the relationship between these two parameters. We will attempt to determine this relationship by simply plotting the experimental results on linear graph paper and then using the resultant curves to suggest their own relationships.

A more or less general equation relating two parameters is the type given as equation (1):

$$y = mx^n + b \qquad (1)$$

Because of its simplicity, we will first consider an equation of this form where the heat flux is the dependent variable and the thermal driving force is the independent variable as in equation (2):

$$q/A = m(\Delta T)^n + b \qquad (2)$$

If the data suggest an equation of this form, we will then attempt to simplify this general equation by determining one or all of the constants. However, we may find that one or all of the constants are affected by parameters such as surface condition or pressure and that equation (2) is already in its most simplified form.

As described below, the above analysis results in the realization that the heat flux is linearly related to the thermal driving force during nucleate boiling. This result disagrees not only with the previous data reduction but also disagrees with muchof the present theory of the boiling process. It will therefore require that much of the present theory be revised to agree with the experimental evidence.

BACKGROUND

In the past thirty years, a great amount of research and analysis has been performed on the subject of boiling. An excellent and recent summary of this large amount of work is presented by Rohsenow in reference (1). With regard to the relationship between heat flux and thermal driving force during nucleate boiling, he states on page 117:

> Although the slope is predominantly in the neighborhood of 3, observations are available with

> resulting slopes of as low as unity for contaminated surfaces and as high as approximately 25 for very clean surfaces (2).

(In the above, the word "slope" refers to the slope of q/A vs. ΔT plotted on log-log graph paper.) On page 125, Rohsenow states

> The great increase in heat transfer rate associated with nucleate boiling $(q/A) \sim (\Delta T)^3$ is due to the agitation . . . .

In a recent discussion of nucleate boiling, Zuber (3) states:

> More than two dozen equations have been heretofore proposed for correlating experimental data . . . .
> All of them can be put in the form
>
> $$h = \text{const.}\ (\Delta T)^m$$
>
> where the value of the exponent varies between 1 and 3 and the . . . . . .

(It should be noted that if the above expression had been written for q/A rather than h, the value of the exponent would have been between 2 and 4.) Zuber also notes that

> . . . variations in the exponent m, ranging from 1 to 25 can be produced by polishing the surface with different grades of emery paper . . .

The above summaries indicate that there is fairly widespread agreement that the heat flux is related to the thermal driving force in a non-linear fashion in which the exponent on the thermal driving force varies over a very wide range but is usually some value around three.

In addition to the above empirical correlations, a number of investigators have taken a theoretical approach to determine the relationship between heat flux and thermal driving force. Examples of theoretical treatments are the analyses of Zuber (3) and that of Levy (4). Levy's analysis results in an equation which relates heat flux to the third power of the thermal driving force. Zuber derives an equation relating heat flux to the thermal driving force raised to some power larger than 5/3. (It is not possible to determine the precise value of Zuber's exponent because his expression includes the surface density of nucleating sites as an empirically measured quantity. Since this surface density is positively correlated with the thermal driving force, his derived exponent is always larger than the exponent he derives for the explicit functionality which is 5/3. The fact that part of the functionality between the two parameters is expressed only implicitly of course places a severe limitation on the usefulness of his equation--i.e. it cannot be used unless one first determines the functionality between this surface density and the other parameters in the system.)

In summary, there has been fairly widespread agreement on both an empirical and theoretical basis that, during nucleate boiling, the heat flux is non-linearly related to the thermal driving force. This non-linearity is such that n in equation (2) would be expected to assume values anywhere from 1 to 25 but would generally be between 2 and 4.

DATA ANALYSIS

The data analysis begins by simply plotting up some experimental results on linear graph paper such as is shown in Figures 1 through 3. Once the points are plotted and the shape of the curves has become more or less apparent, we must then try to determine that relationship which seems most strongly suggested by the data. For the ten "runs" shown in Figures 1 through 3, this is an extremely simple matter because the nucleate boiling portions of the curves come extremely close to being straight lines which immediately suggests the relationship of equation (2) wherein the exponent n is equal to 1.0. Thus, all ten of the runs strongly suggest that the heat flux is linearly related to the thermal driving force by the relationship

$$q/A = m(\Delta T)^{1.0} + b \qquad (3)$$

The fact that the several runs exhibit various values for m and b suggests that these constants are not pure constants and that they are indeed functions of such variables as surface condition, pressure, and the many other variables encountered in boiling systems.

Close inspection of Figures 1 through 3 reveals that, of the 62 nucleate boiling points plotted, virtually every one falls within 2 F. and that the average deviation from the straight lines is less than 1 F. This amazing precision strongly demonstrates that there is indeed no non-linearity to the data shown in the Figures. Moreover, it should be noted that the data in the figures is a fairly widespread sample of nucleate boiling data--i.e. the data consist of runs obtained

by two different investigators

for two different boilers

for three different fluids

for four different pressures

for five different surface finishes

The fact that every run exhibits a linear relationship between the variables in spite of the widely different nature of the runs strongly demonstrates that this linear relationship is generally applicable. It is not peculiar to a certain type or set of conditions because we have examined a number of different types of conditions and they have all exhibited a very high degree of linearity. We are thus led to conclude that equation (3) applies in general to nucleate boiling data in spite of the fact that there is widespread agreement that the exponent in the equation should exhibit some value greater than 1.0 and more like three.

It may perhaps be more convincing to observe that the data in Figures 1 through 3 would give reasonably straight lines if plotted on log-log paper and that the slopes would indeed be in the neighborhood of three or four. For instance, the data in Figure 3 is presented by Rohsenow on page 119 of reference (1) as a log-log graph in which straight lines are drawn through the nucleate boiling data points. On this particular graph, the slope varies from about two to six, seeming to suggest that the relationship is non-linear in spite of the fact that it is highly linear as shown in Figure 3. The reason for this seeming anomaly is presented below.

LOG-LOG GRAPH PAPER

It was mentioned above that the use of log-log graph paper involves a tacit assumption about the relationship between the variables of interest. This may be seen by noting that equation (1) will be a straight line on linear graph paper only if $n = 1.0$ and will be a straight line on log-log graph paper only if $b = 0.0$. Thus, when we attempt to draw a straight line through data on log-log graph paper, we tacitly assume that b does indeed equal zero. Thus, our result will be quantitatively correct only if we verify the accuracy of this tacit assumption. This of course is easily done by plotting the data on linear graph paper and extrapolating the data to determine the value of b. If b does not equal zero, the data would indicate the wrong value of the exponent if it were plotted directly on log-log graph paper and the slope were then measured. This is precisely why the data in Figure 3 indicate an exponent of 1.0 on linear graph paper and an exponent of 2 to 6 when plotted on log-log paper. As a result of the finite value of b in equation (2), the exponents deduced from the slope on log-log paper are incorrect. Even a cursory glance at Figures 1 through 3 will indicate that b is indeed finite (as one would expect by virtue of the fact that boiling requires a finite thermal driving force).

The above is not to say that one is helpless in the face of a finite value of b. In such a case, the variables would simply require a minor transformation so that the transformed equation would exhibit a zero value for b. After transforming the data, the log-log graph would indeed exhibit a slope which was in agreement with the exponent desired. If the reader will perform this transformation on any of the runs shown in Figures 1 through 3 and then plot the transformed data on log-log paper, he will find that the resultant slope is indeed 1.0.

The importance of first performing this transformation before attempting to draw straight lines on log log graph paper is best illustrated by noticing that an exponent of as high as 25 can be deduced for a set of data in which the exponent is truly 1.0. The safest way to avoid this type of error would seem to be to always plot the data first on linear graph paper.

RESULTS

The obvious result of the above is the realization that heat flux and thermal driving force are linearly related during nucleate boiling, and that this result is generally applicable. This will in turn simplify the correlation of this type of data and will somewhat simplify the design of boiling equipment in the sense that it is more convenient to work with linear correlations than with non-linear ones.

A less obvious but far more important result is the fact that this linear relationship is in violent disagreement with much of our present theory about the boiling process. As a result of this disagreement, much of the present theory will have to be revised in order to agree with the experimental evidence. For instance, the separate analyses of Zuber and of Levy both resulted in a non-linear relationship between heat flux and thermal driving force. Since this disagrees with the experimental evidence, it is manifest that the theory on which these analyses were based was incorrect. Moreover, it seems reasonable to expect that this new understanding of the relationship between the heat flux and the thermal driving force will itself suggest much of the new theory required to resolve the discrepancy between theory and measurement. For instance, the fact that Berenson's data is linear without break right up to the maximum heat flux would seem to suggest that there is only one regime of nucleate boiling (instead of the two proposed by Zuber's theory of bubble interference presented in reference (3)). Since this discussion of the theory is a digression from the principle purpose of this article, it would seem preferable to defer it to a later article.

Conclusions

The major conclusions resulting from the above analysis are:

1. During nucleate boiling, the heat flux and the thermal driving force are linearly related.
2. The non-linearity which seemed to result from many previous analyses was a result of the vagaries of log-log graph paper.
3. Much of the present theory of the boiling process is invalidated by the fact that the heat flux and the thermal driving force are shown to be linearly related in direct contradiction to the theory.
4. The understanding of the linear relationship should result in an improved understanding of the boiling process.

SYMBOLS

A area

b dimensional constant

h heat transfer coefficient

m dimensional constant (dimensionless as used by Zuber)

n dimensionless constant

q heat

T temperature

x unspecified variable

y unspecified variable

References

1. Modern Developments in Heat Transfer by Ibele, Heat Transfer with Boiling by Warren M. Rohsenow (1963)

2. McAdams, W. H., Heat Transmission, 2nd edition, p. 322 (1954)

3. Zuber, Novak, Nucleate Boiling. The Region of Isolated Bubbles and the Similarity with Natural Convection, Int. J. Heat Mass Transfer, Vol. 6, pp. 53-78

4. Levy, S., Generalized Correlation of Boiling Heat Transfer, ASME Transactions, Series C (1960)

5. Cichelli, M. T. and Bonilla, C. F., Trans. A. I. Ch. E. 41, 755-787 (1945)

6. Berenson, P., Transition boiling heat transfer from a horizontal surface, Sc. D. Thesis, Mech. Eng. Dept., Mass. Inst. Technol. (1960)

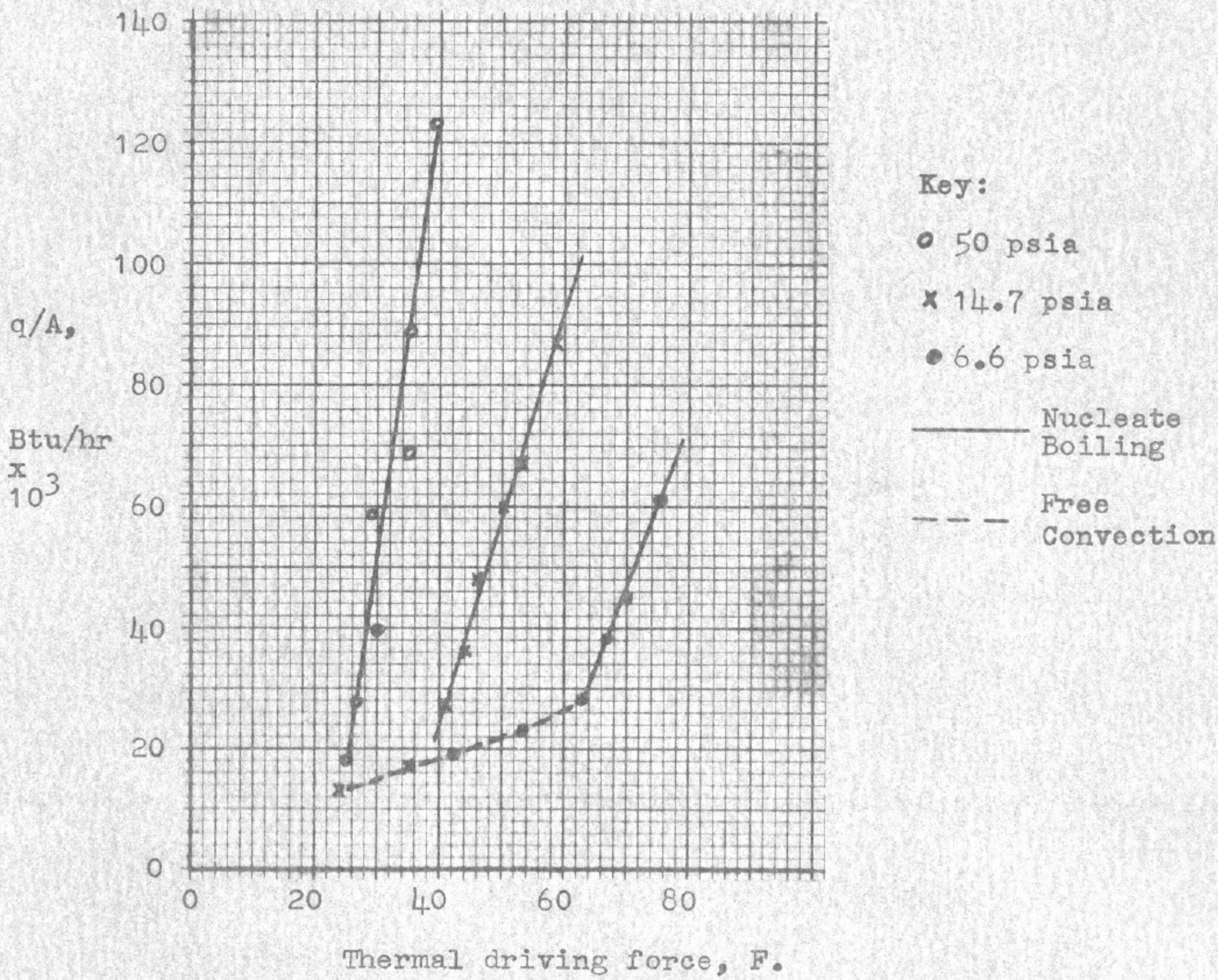

FIGURE 1

Heat Flux vs. Thermal Driving Force, n-Heptane, at Several Pressures (Reference 5 data, Cichelli & Bonilla)

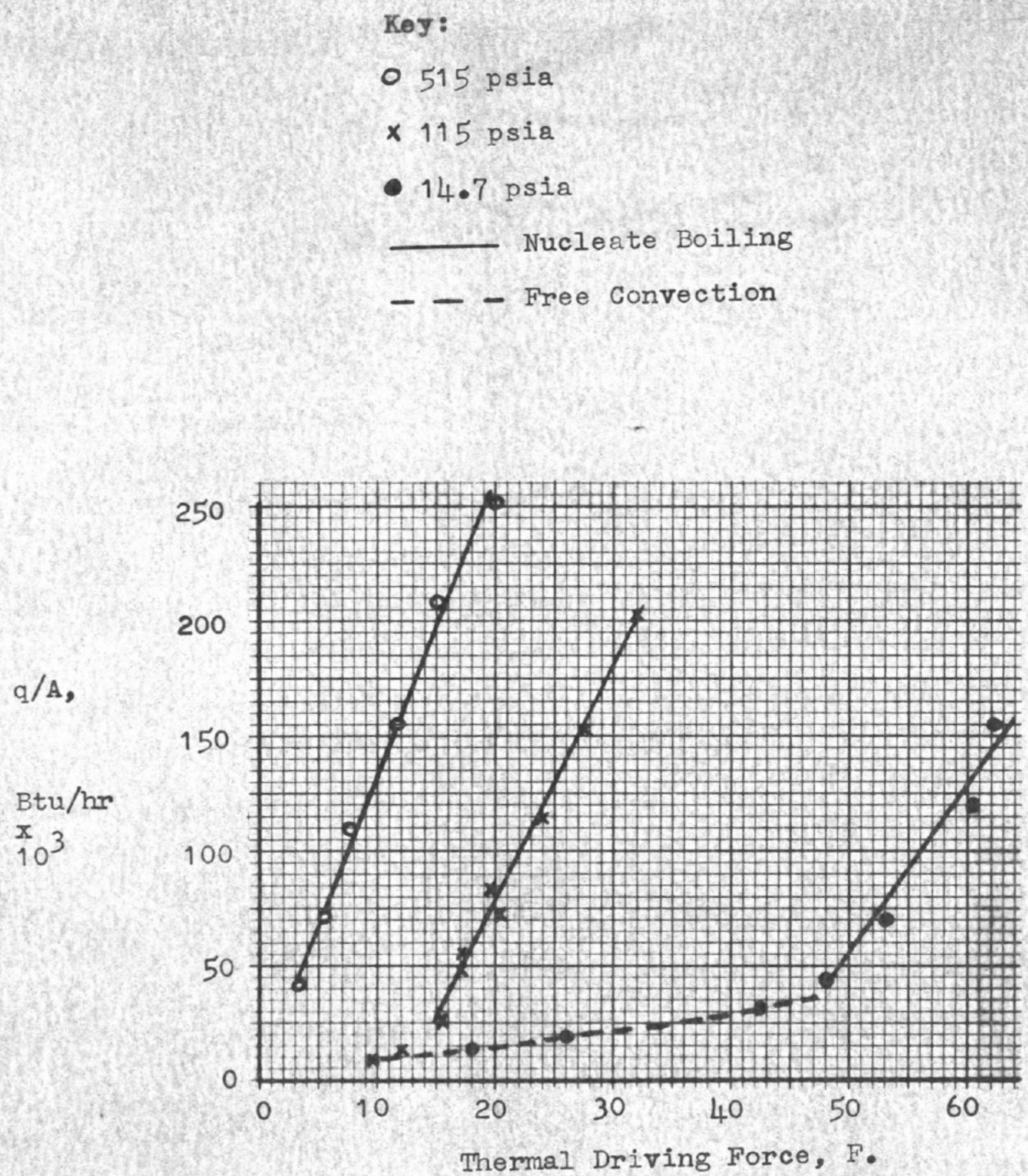

FIGURE 2

Heat Flux vs. Thermal Driving Force, Ethyl Alcohol, at Several Pressures (Reference 5 data, Cichelli & Bonilla)

Key:

● Run 31: Emery 320 OK 20°

□ Run 32: Emery 60 12°

▲ Runs 17 & 22: Lap E OK 8°

× Run 2: Mirror Finish 24°

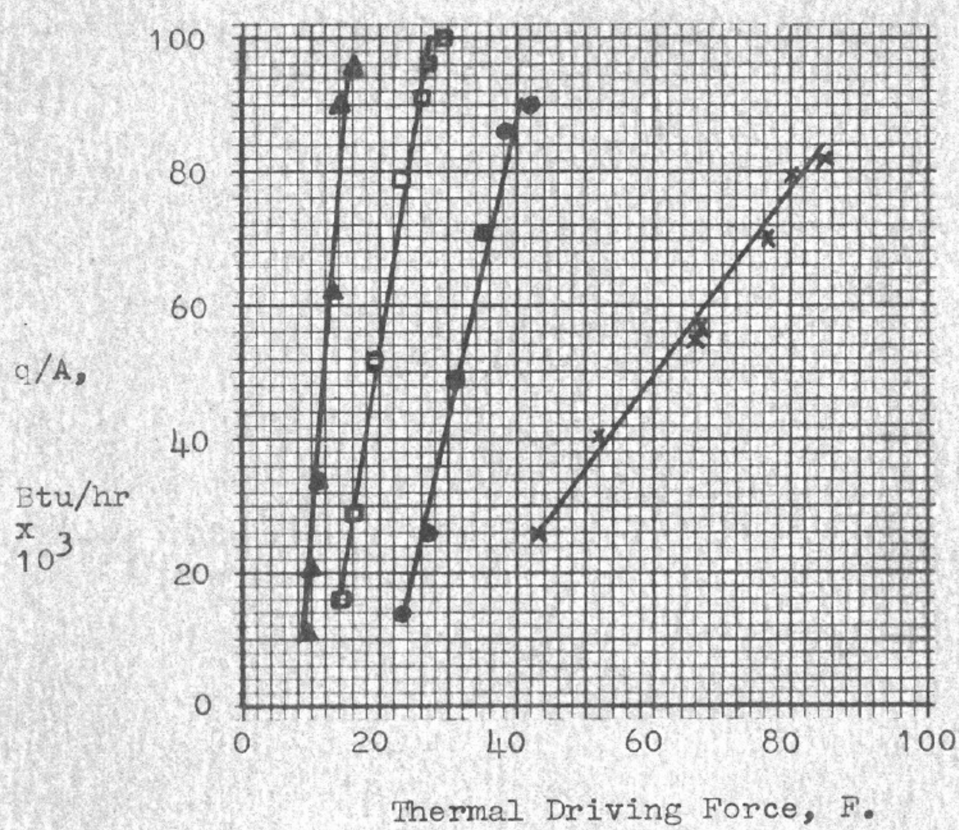

FIGURE 3

Heat Flux vs. Thermal Driving Force, n-pentane, for various Surface Finishes (Reference 6 data, Berenson)