September 15, 1963

Harold L. Davis
Associate Editor
N ucleonics
330 West 42 Street
New York 36, N. Y.

Dear Harold,

Enclosed is the first article in the series of two we have disc ussed recently. You mentioned a length of 3-4 pages per article and I think the attached article should just fit in 4 pages. However, if it is too long for 4 pages and you cannot allow 5 pages, the section on "Background of Hydraulic Stability" can be omitted without affecting the continuity. If you decide to omit this section, please consider this letter a request for a 3 page article on Hydraulic Stability. The subject section is painfully brief as it is, and presenting a 3 page article under a different title may be more appropriate.

In your editorial introduction to the subject series, I would appreciate your pointing out the following:

1. Space limitations have not allowed an adequate treatment of the subject of thermal stability.

2. I am presently planning to present a paper of much broader scope on stability in general and thermal stability in particular at the spring meeting of the ASME.

3. Stability is one of the many subjects covered in a book entitled "Boiling" which I am presently writing and which should be published sometime in 1964.

With regard to publishing my book on Boiling, I would very much like to have McGraw-Hill publish it. Dr. Bonilla was to speak with someone at M-H about publishing my book, but I have not heard from him recently. Please advise as to whom I should contact at M-H and how favorably you would recommend my book on the basis of the attached article. My book will bring as much originality and simplification to the subject of boiling in general as the attached article brings to the subject of stability.

Thank you.

Sincerely yours,

Eugene F. Adiutori

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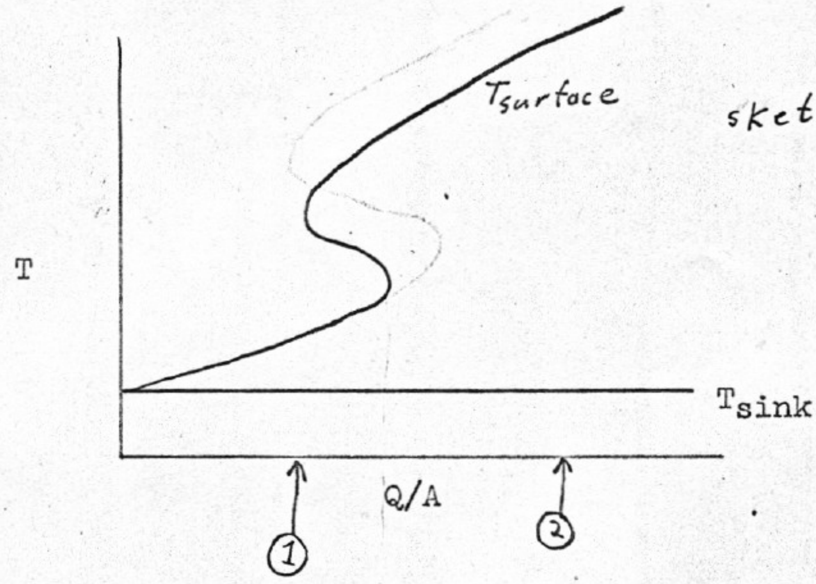
# THERMAL STABILITY, Part I

by

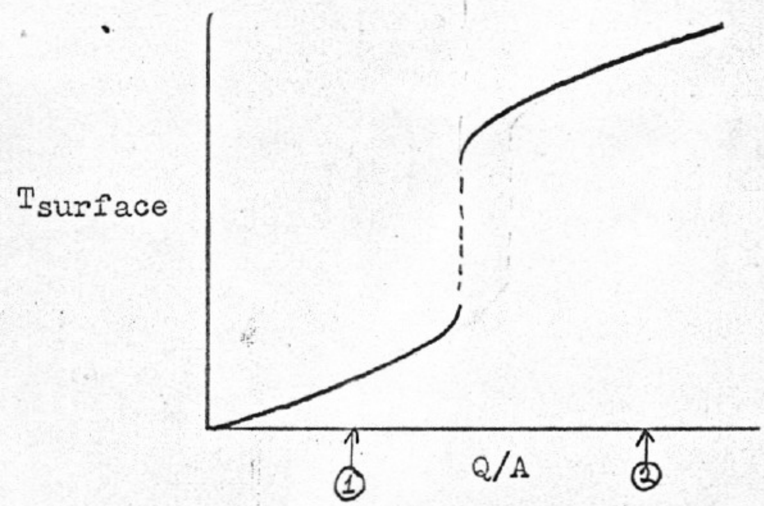
Eugene F. Adiutori

## Introduction:

This is the first of two articles to be presented on the subject of thermal stability. The importance of this subject has long been recognized as evidenced by the large amount of data presented in the literature on the subject of "burnout" or "critical heat flux" in two-phase systems. The existence of thermal instability is often demonstrated graphically by observing that, if the heat flux in sketch (a) is monotonically inc reased from 1 to 2, the surface temperature is a discontinuous function of heat flux as shown in sketch (b):




sketch (a)




sketch (b)

The difficulty with this graphical approach is that, while the above result is reasonably accurate in many systems, the method can also lead to results which are far from accurate. For instance, the graphical approach can lead to the erroneous conclusion that any point on sketch (a) can be obtained by designing the equipment so that the temperature of the heat sourc e is controlled. That this is not ~~necessary~~ necessarily so can be seen by observing that the temperature of the heat source is never* equal to the surface temperature in sketch (a) b ecause there is always a finite thermal resistance between the heat source and the surface. Thus, controlling the temperature of the heat source does not necessarily result in a stable surface temperature.

Another difficulty with the graphical approach is that, to date, it has not led to the valid conclusion that systems can oscillate as a result of thermal instability.

Boiling liquid metal systems exhibit the requirements for thermal instability to a larger degree than any other class of systems. With the advent of liquid metal boilers for the space program, thermal stability becomes a problem of great practical importance. However, it is extremely important to note that all heat transfer systems must possess thermal stability in order to operate in a non-oscillatory manner.

## General Definition of Stability:

Definitions are often treated as a "necessary evil" which is required to enable intelligent communication. However, a good definition will oftentimes suggest a method of analysis. With this in mind, a good definition of stability would seem to be:

> A parameter (T) is stable at a particular value ($T_{iv}$) provided that, when $T_{iv}$ is perturbed to ($T_{iv} + \Delta T$), equation 1 is satisfied.
>
> $$T(\ t=\infty) = T_{iv} \qquad (1)$$
>
> If the result of the perturbation does not satisfy equation (1), the system is unstable.

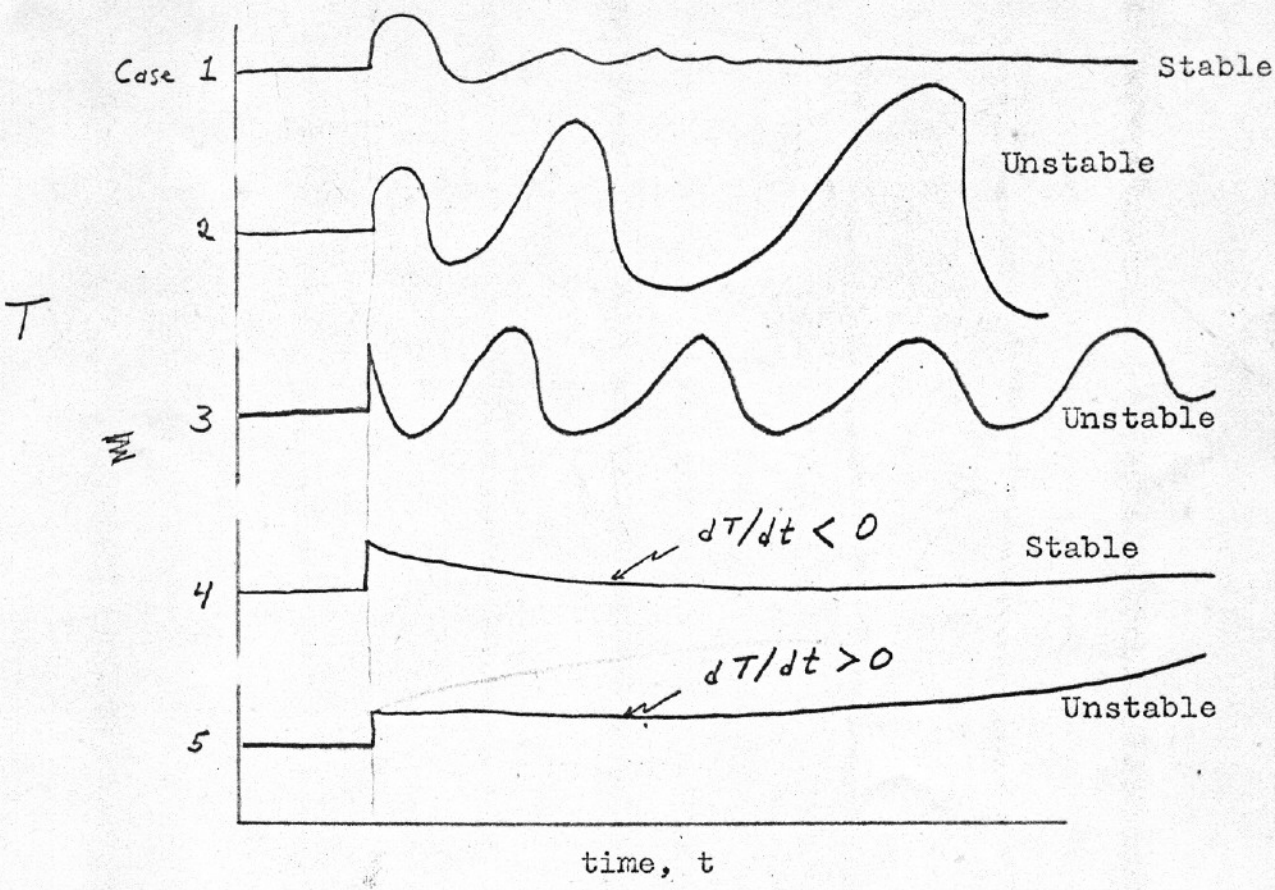
The above definition of course strongly suggests that stability be appraised by perturbing the system and observing its response.

## Simplifying Stability Analyses:

If a given system is perturbed, it may respond in any of the ways shown in sketch (c).

* Except at the singular point Q/A = 0.

sketch (c)




It should be noted in sketch (c) that, if the system being analyzed behaves like Cases 4 and 5 ( i.e. dT/dt is either greater or less than zero everywhere in the interval $0<t<\infty$), the stability can be easily appraised by determining dT/dt at $t=0+\Delta t$. For these idealized cases, the stability criteria become:

1. The system is stable if dT/dt is opposite in sign to $\Delta T$.

2. The system is unstable if dT/dt is of the same sign as $\Delta T$.

3. The system is unstable if $dT/dt = 0$.

(All the above derivatives may be evaluated anywhere in the interval $0<t<\infty$).

It is indeed unfortunate that real systems do not necessarily behave like Cases 4 or 5. However, instability can be most easily analyzed by idealizing real systems in such a way that they behave exactly like Cases 4 and 5. After solving the idealized problem, it then becomes an easy matter to remove the idealized aspects and solve the real, physical problem.

It will be noted that the above approach is followed throughout both of these articles. Without this simplification, we would almost certainly become bogged down with the period and amplitude of the oscillations which occur in the interval $0<t<\infty$ and which would certainly not help us appreciate or identify the problem.

Background of Hydraulic Stability:

Since the analysis of hydraulic stability has preceded that of thermal stability, and since mass and heat transfer are so analagous, it will be helpful to briefly review the status of hydraulic stability.

An early paper on the subject of hydraulic stability was that of Ledinegg (1), published in 1938. As described by Anderson and Lottes (2), Ledinegg derived an expression for two phase pressure drop and "defined" the criterion for hydraulic stability to be

$$d\Delta P/dW \leq 0 \quad (2)$$

Using the derived expression ~~and equation (2)~~, Ledinegg showed that equation (2) could be rewritten as

$$\Delta h \geq \frac{BD}{B+\frac{1}{g}-\left(\frac{3}{2}B\left[\frac{B}{2}+\frac{1}{g}\right]\right)^{1/2}} \quad (3)$$

It was apparently first noted by Dr. W. H. Jens that Ledinegg's "definition" of hydraulic instability was, in reality, an assumption about the performance characteristics of the system being analyzed. Jens noted that the definition would apply only if the system were such that the "available" pressure drop were independent of the system flow rate. Clark and Rohsenow state (3):

> The design of boiling systems which are stable requires the use of a pump which puts out a constant-flow rate, or if a centrifugal pump is used, that the boiling section pressure drop be a small proportion of total pressure drop. This is suggested by Dr. Jens. . . . The significant point here is that nowhere does the system $\Delta P$ characteristic become tangent to the pump characteristic.

Mathematically, the last sentence is equivalent to equation (4):

$$d\Delta P_{pump}/dW < d\Delta P_{test\ section}/dW \quad (4)$$

This author would like to add that, in the assumed system, equation (4) is necessary but <u>not</u> sufficient and that the system could be unstable even if

$$d\Delta P_{pump}/dW = -\infty \qquad (5)$$

It can be shown that equation (4) is sufficient only for systems which behave like Cases 4 and 5 of sketch (a). That equation (4) is not sufficient can be recognized by noting that, if there is a compressible accumulator between the pump and the test section, a constant flow rate pump would imply only that

$$\overline{W_{pump}} = \overline{W_{test\ section}} \qquad (6)$$

$$W_{pump}(t) = W_{accumulator}(t) + W_{test\ section}(t) \qquad (7)$$

$$= \text{constant}$$

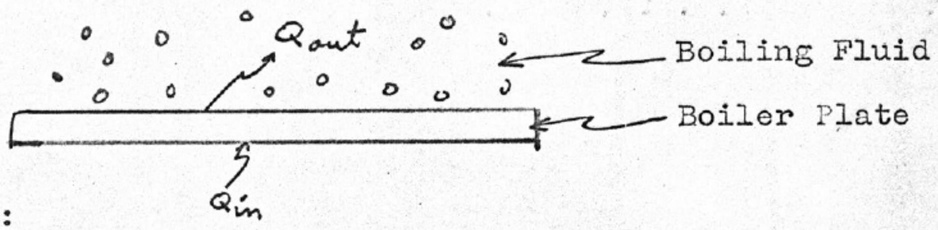
Equation (7) obviates the conclusion that the test section flow rate would necessarily be constant if the pump flow rate were constant.

That equation (4) is not sufficient was graphically illustrated by the results of Aladyev (4) in which an accumulator was intentionally placed between the pump and the test section. The compressible accumulator reduced the critical heat flux by as much as a factor of five for identical inlet conditions, conceivably due to the effect of the accumulator on the hydraulic stability of the test section. Aladyev's results also demonstrated that hydraulic instability can lead to flow oscillations as well as flow discontinuities.

Thermal Stability in an Idealized System:

The above discussion was partly intended to illustrate that stability criteria can be simply stated only for very simple systems. The stability criteria for real systems are much more c omplex and require that inertia, compressibility, and storage be considered. Toward the end of obtaining a simple criterion for thermal stability, let us now analyze a very idealized system. However, we must bear in mind that the resultant criterion will be applicable only to systems which closely resemble the idealized system described below:

sketch (d)




Bases:

1. The temperature of the boiling fluid is constant-- i. e. is unaffected by $Q_{out}$.

2. The heat capacity of the boiler plate is finite.

3. The thermal conductivity of the boiler plate is infinite.

4. $Q_{in}$ is constant--i.e. is unaffected by the temperature of the boiler plate.

In the above idealized system, the parameter we wish to investigate is the temperature of the boiler plate. As suggested by the definition of stability, we shall perturb the temperature and determine under what conditions the temperature is stable. The analysis is based on noting that

$$Q_{in}(t) - Q_{out}(t) = Q_{stored}(t) \tag{8}$$

$$Q_{stored,\ iv}(t=0) = 0 \tag{9}$$

When the boiler plate temperature is perturbed (a differential amount) from some initial value at which equation 9 is satisfied,

$$Q_{out}(t) = Q_{in} + \left(\frac{d\,Q_{out}}{d\,T_w}\right)\left(\Delta T_w(t)\right) \tag{10}$$

$$Q_{in}(t) = Q_{in} + \left(\frac{d\,Q_{in}}{d\,T_w}\right)\left(\Delta T_w(t)\right) = Q_{in} \;\star \tag{11}$$

Therefore, from 8, 9, 10, and 11,

$$Q_{stored}(t) = -\left(\frac{d\,Q_{out}}{d\,T_w}\right)\left(\Delta T_w(t)\right) \tag{12}$$

Now,

$$\left(\frac{d\,T_w}{dt}\right) = \frac{Q_{stored}(t)}{C_{Boiler\ Plate}} \tag{13}$$

Therefore, from (12) and (13),

$$\left(\frac{d\,T_w}{dt}\right) = -\left(\frac{1}{C}\right)\left(\frac{d\,Q_{out}}{d\,T_w}\right)\left(\Delta T_w(t)\right) \tag{14}$$

$\star$ Since $\dfrac{d\,Q_{in}}{d\,T_w} \equiv 0.$

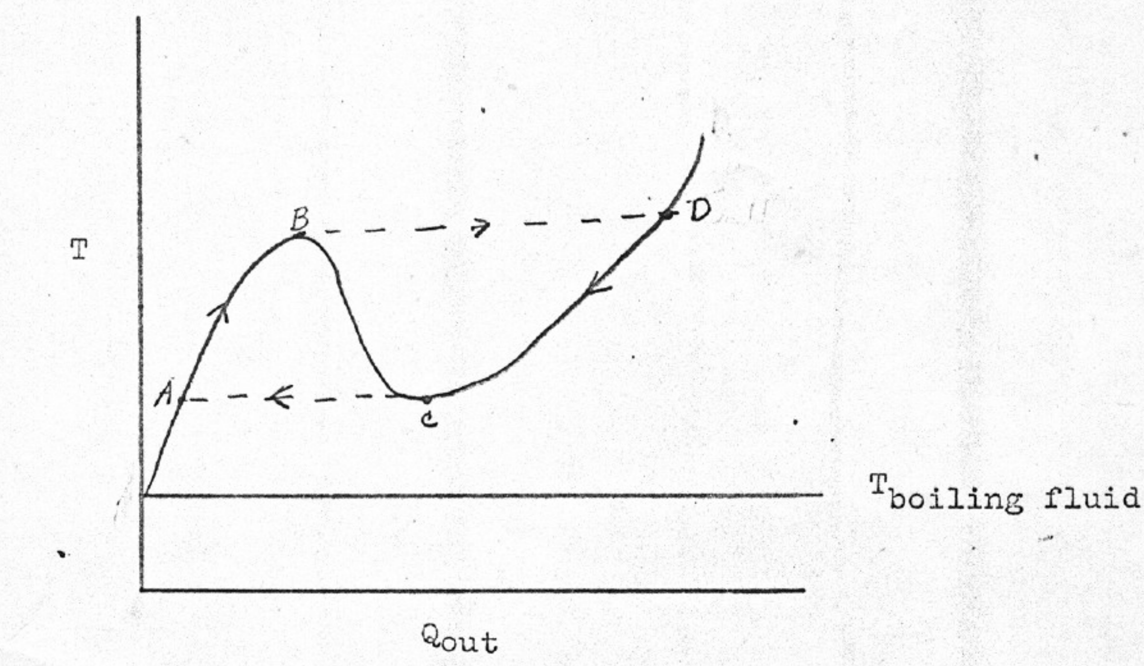
From equation (14), it can be seen that $T_{wall,iv}$ will be stable only if

$$\left(\frac{dQ_{out}}{dT_W}\right)_{T_W,iv} \underset{s}{>} 0 \qquad (15)$$

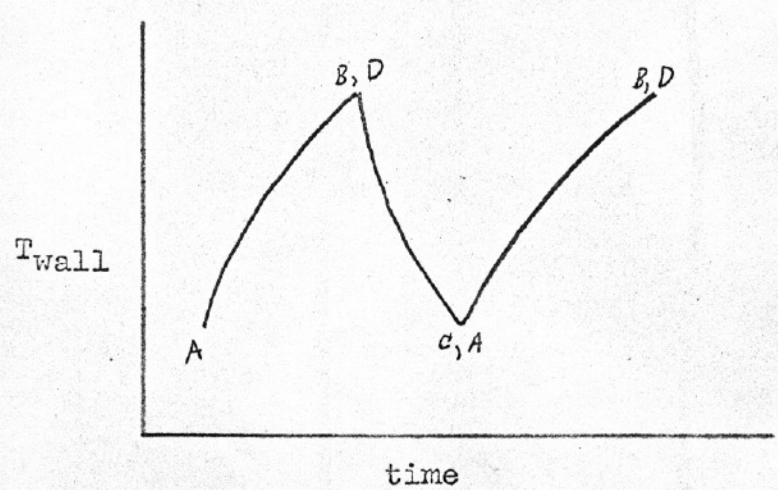
since this the only condition under which $T_{wall}$ (t) will tend toward $T_{wall,iv}$.

Equation (15) is the stability criterion for the idealized system and is remarkably similar to its hydraulic counterpart. The criterion states that the boiler plate temperature would be unstable everywhere in interval B C of sketch (e). Moreover, if $Q_{in}$ is set anywhere in interval BC, the system will oscillate as shown in sketches (e), (f), and (g).

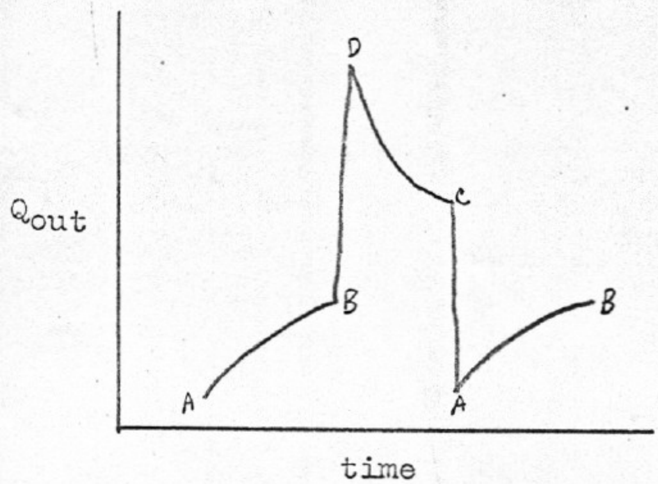
sketch (e)




sketch (f)



sketch (g)

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The main conclusion to be drawn from the above is that, in a constant heat input system, only the heat <u>input</u> is necessarily invariant with time.

The operator has <u>no</u> control over $\underline{Q_{out}}$ vs. time - this must be determined by the <u>system</u>.

Another conclusion to be drawn from the above is that, had our system been a forced flow system in which the flow was a function of void fraction in the channel, the <u>flow rate</u> would also <u>oscillate</u> as a result of the thermal instability!

In closing, it should be emphasized that equation (15) applies <u>only</u> to the idealized system. However, if a real system approximates the idealized system, equation (15) should predict the onset of instability with reasonable accuracy. On the other hand, if the real system differs markedly from the idealized system, very poor prediction should be expected. For instance, if the temperature of the heat source is controlled in the real system, equation (15) should be disregarded and a different idealized system analyzed. In this case, the stability analysis proceeds exactly as outlined above, although the result is considerably different.

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The next article in this series will present examples of thermally unstable systems taken from the literature. It will also briefly discuss the manner in which thermal instability can lead to flow oscillations, premature burnout, and reactor power oscillations. In closing, it will suggest avenues by which thermal stability in real systems can be improved.

Nomenclature:

symbols

| | |
|---|---|
| L | length |
| T | temperature |
| Q | heat per unit time (always taken as positive) |
| A | area |
| t | time |
| $\Delta P$ | pressure drop |
| f | friction factor |
| D | equivalent diameter |
| W | flow rate |
| h | enthalpy |
| h | $(h_{sat} - h_{inlet})$ where $h_{inlet} < h_{sat}$ |
| v | specific volume |
| $\rho$ | density |
| B | $fL/2gD$ |
| D | $(v_f / v_{fg})\, h_{fg}$ |
| C | total heat capacity, Btu/oF |
| $<_u$ | indicates that the system is <u>un</u>stable if the equation is satisfied |
| $<_s$ | stable, similar to above |

Subscripts

| | |
|---|---|
| sat | refers to saturation |
| f | " " liquid |
| fg | " " phase change, liquid to gas |
| g | " " gas |
| iv | " " initial value |
| w | " " wall |

References:

(1) M. Ledinegg, Unstabilität der Strömung bei natürlichem und Zwangsumlauf, Die Wärme, 891 - 898 (1938)

(2) R. P. Anderson and P. A. Lottes, Boiling Stability, reprinted from Progress in Nuclear Energy, Series IV, Vol. 4, Technology, Engineering, and Safety, (1961)

(3) J. A. Clark and W. M. Rohsenow, Local Boiling Heat Transfer to Water at Low Reynolds Numbers and High Pressures (Author's Closure), ASME Transactions, Vol. 76, pp. 553 - 562, (1954)

(4) I. T. Aladyev, et al, Boiling Crisis in Tubes, 1961 International Heat Transfer Conference at Boulder, Colorado, Vol. II pp 237 - 243