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A NEW VIEW OF DIMENSIONAL HOMOGENEITY, AND ITS IMPACT ON THE FUNDAMENTAL EQUATIONS AND PARAMETERS OF HEAT TRANSFER SCIENCE

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ABSTRACT

A new view of dimensional homogeneity is presented which observes that Natural phenomena generally exhibit inhomogeneous behavior, and therefore correlations which describe Natural phenomena should be inhomogeneous.

Application of this new view to heat transfer science results in the conclusion that "Newton's Law of Cooling" and "Fourier's Law" should be abandoned in favor of Eqs (1) and (2). These inhomogeneous equations state that heat flux (q) is a function of temperature difference (convection) or temperature gradient (conduction).

$$q = f(\Delta T) \qquad (1)$$

$$q = f(dT/dx) \qquad (2)$$

NOMENCLATURE

a	constant coefficient, dimensionless
d	diameter, m
h	heat transfer coefficient, kW/m^2C
I	electric current, amps
k	thermal conductivity, kW/mC
P	pressure, N/m^2
q	heat flux, kW/m^2
s	distance, m
t	time, s
T	temperature, C
V	potential difference, volts, or velocity, m/s
W	flow rate, kg/s
x	distance, m
ϵ	strain, dimensionless
σ	stress, N/m^2

INTRODUCTION

Dimensional homogeneity dictates that only quantities with identical dimension may be added/subtracted. Apples may not be added to oranges--feet may not be added to seconds--pounds may not be subtracted from inches. Homogeneity also dictates that each term in an equation have identical dimension. Otherwise an equation could call for the addition/subtraction of quantitities of different dimension.

Science has required dimensional homogeneity for more than 2000 years. Even so, it is appropriate to ask:

- Do Natural phenomena generally exhibit homogeneous behavior? In other words, do Natural phenomena behave in a way which is rigorously described only by equations which are dimensionally homogeneous?
- If Natural phenomena do not generally exhibit homogeneous behavior, is it necessary to describe Natural phenomena with homogeneous equations?
- If Natural phenomena do not generally exhibit homogeneous behavior, is it necessary to retain the parameters invented to achieve homogeneity?
- If homogeneous Laws and the parameters invented to achieve homogeneity are abandoned, what will replace them?

This article answers these questions in a general way, and in a specific way with regard to heat transfer science.

THE ANCIENT GREEK VIEW OF DIMENSIONAL HOMOGENEITY

Dimensional homogeneity is not unique to modern science. In varying form, it has been an integral part of science since the days of Aristotle.

The ancient Greek view of dimensional homogeneity is described in the following by Drake (1974):

> *Algebraically, speed is now represented by a "ratio" of space traveled to time elapsed. For Euclid and Galileo, however, no true ratio could exist at all except between two magnitudes of the same kind.*

Similarly, Cohen (1985) states:

> *Aristotle and most early scientists, including Galileo, preferred to compare speeds to speeds, forces to forces, and resistances to resistances.*

The above statements indicate that ancient Greek science required dimensional homogeneity *within each term*--ie the numerator and denominator of each term were required to have identical dimension. For example, distance could not be divided by time because (distance/time) is not homogeneous. Distance could be divided only by distance, and time only by time.

The following by Galileo (1638) indicates that he did in fact share the ancient Greek view of homogeneity:

> *If two particles are carried at a uniform rate, the ratio of their speeds will be the product of the ratio of the distances traversed by the inverse ratio of the time-intervals occupied.*

Notice that each term in Galileo's equation contains the same dimension in numerator and denominator. The terms are the ratio of speed to speed, distance to distance, and time to time.

Also notice that Galileo saw no conflict between the concept of speed and the view that distance cannot be divided by time. To Galileo, distance and time were necessary to quantify speed, but speed had nothing to do with *dividing* distance by time.

FOURIER'S (1822) VIEW OF DIMENSIONAL HOMOGENEITY

Maxwell (1891) states: *"The theory of dimensions was first stated by Fourier, Theorie de Chaleur".*

Fourier's contemporaries (Biot for example) were unable to cope with conductive heat transfer because they did not know that the flow of heat depends on the length and area of the heat flow path. Thus they attempted to deal with conductive heat flow in a general way using equations which were dimensionally *IN*homogeneous.

Fourier (1822) notes the inhomogeneity in the equations of his contemporaries, and points out its disastrous effect on their ability to deal with heat transfer phenomena in a general way:

> *(The flow of heat depends on the path length.) We lay stress on this remark because the neglect of it has been the first obstacle to the establishment of the theory. If we did not make a complete analysis of the elements of the problem, we should obtain an equation not homogeneous and, a fortiori, we should not be able to form the equations which express the movement of heat in more complex cases.*
>
> *It was necessary also to introduce into the calculation the dimensions of the prism, in order that we might not regard, as general, consequences which observation had furnished in a particular case. Thus, it was discovered by experiment that a bar of iron, heated at one extremity, could not acquire, at a distance of six feet from the source, a temperature of one degree; for to produce this effect, it would be necessary for the heat of the source to surpass considerably the point of fusion of iron; but this result depends on the thickness of the prism employed. If it had been greater, the heat would have been propagated to a greater distance . .* (Article 75).

(The second paragraph refers to an article by Biot published in 1804. Because Biot did not know that the conduction of heat depends on the area of the flow path, he erroneously generalized that, six feet from the heated end, the temperature of *ANY* solid iron bar would be less than one degree above ambient because the bar would melt before the temperature six feet from the heated end reached one degree. Notice the confident and offhand way in which Fourier corrects Biot's error.)

Fourier describes his view of dimensions, dimensional homogeneity, and dimensional analysis in the following:

> *. . . every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimensions. We have introduced this consideration into the theory of heat, in order to make our definitions more exact, and to serve to verify the analysis; it is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us without proof.* (Article 160)

Notice that Fourier *introduces* homogeneity into the theory of heat in order *"to make our definitions more exact, and to serve to verify the analysis".* Notice that Fourier does not *prove* that homogeneity is necessary. He simply states *"it is derived from primary notions on quantities (and) . . . is the equivalent of the fundamental lemmas (axioms)"* which are also accepted without proof.

THE MODERN VIEW OF DIMENSIONAL HOMOGENEITY

The modern view of dimensional homogeneity is essentially the same as that of Fourier. Langhaar (1951) describes the modern view:

> *An equation will be said to be dimensionally homogeneous if the form of the equation does not depend on the fundamental units of measurement. For example, the equation for the period of oscillation of a simple pendulum*
>
> $$T = 2\pi(L/g)^{.5} \qquad (1)$$
>
> *is valid whether length is measured in feet, meters, or miles, and whether time is measured in minutes, days, or seconds. Therefore, by definition, the equation is dimensionally homogeneous. If the value $g=32.2$ ft/sec^2 is substituted in the equation there results*
>
> $$T = 1.11\ L^{.5} \qquad (2)$$
>
> *This equation is correct for pendulums on earth, but it is no longer dimensionally homogeneous, since the factor 1.11 applies only if length is measured in feet and time is measured in seconds. It might be argued that the factor 1.11 itself has the dimension (which would make the equation homogeneous). However, dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*
>
> *It can be deduced from the above definition of dimensional homogeneity that an equation of the form $x = a + b + c + \ldots$ is dimensionally homogeneous*

if, and only if, the variables x, a, b, c . . . all have the same dimension. . . If a derived equation contains a sum or a difference of two terms that have different dimensions, A MISTAKE HAS BEEN MADE.

White (1979) describes the modern view:

(Dimensional homogeneity) is almost a self-evident axiom in physics. . . (It) can be stated as follows: If an equation truly expresses a proper relationship between variables in a physical process, it will be dimensionally homogeneous; ie each of its additive terms will have the same dimension.

THE MODERN VIEW VS THE ANCIENT GREEK VIEW

The modern view of dimensional homogeneity is a modification of the ancient Greek view. The passages cited above from Drake (1974) and Cohen (1985) indicate that the Greek view is summarized as follows:

Dimensions may NOT be added/subtracted/multiplied/-divided.

Dimensioned quantitities may be added/subtracted/-divided ONLY if their dimensions are identical.

The modern view is summarized as follows:

Dimensions MAY be multiplied/divided. They may NOT be added/subtracted.

Dimensioned quantities MAY be multiplied/divided even if they are of unlike dimension. They may be added/subtracted ONLY if they are of like dimension.

Notice that the Greek view is consistent in that all mathematical operations on dimensions are forbidden. Also note that the modern view is inconsistent in that it forbids addition/subtraction of dimensions, but allows multiplication/division.

HOW TO DETERMINE WHETHER NATURAL PHENOMENA ARE GENERALLY HOMOGENEOUS

In order to determine whether Natural phenomena are generally homogeneous, we have merely to examine Natural behavior. If we find evidence of homogeneous behavior, we may accept the current view of homogeneity. But if we find no evidence of homogeneous behavior, then we must conclude that:

- Natural phenomena do *NOT* generally exhibit homogeneous behavior.
- There is no foundation to the current view that Natural phenomena are rigorously and scientifically described only by homogeneous equations.

A ONE PHASE, FORCED CONVECTION EXPERIMENT BY FOURIER

Let us search for homogeneity in data from a heat transfer experiment similar to one performed by Fourier--a one phase, forced convection experiment. Fourier (1822) does not present the data, but he describes the experiment in the following:

Suppose that a body bounded by a plane surface having a certain area (a square metre) is maintained . . . at constant temperature 1, common to all its points, and that the surface . . . is in contact with air maintained at temperature 0: . . . thus a certain quantity of heat denoted by h will flow through the surface in a definite time (a minute).

The air is supposed to be continually displaced with a given uniform velocity . . .

The result of observation is . . . that this quantity of heat lost may be regarded as sensibly proportional to the excess of the temperature of the body over that of the air and surrounding bodies, . . . all the other circumstances remaining the same. (Articles 30 to 31.)

Notice that the principal result of Fourier's experiment was the observed proportionality between q and ΔT--ie the data indicated that q{ΔT} is a proportional function for one phase, forced convection heat transfer.

An experiment such as the one described by Fourier would include the following data: heat transferred; heat transfer area; coolant flow rate; coolant inlet and outlet temperatures; temperature of the heat transfer surface. The reduced data would be a set of heat fluxes and temperature differences similar to the data set listed in Table 1.

TABLE 1

HEAT TRANSFER DATA SET

Heat Flux, kW/m^2	ΔT, C
1.0	4.7
2.0	9.4
3.0	14.1
4.0	18.8
5.0	23.5
6.0	28.2
8.0	37.6
10.0	47.0

HOW FOURIER ACHIEVED HOMOGENEITY

The manner in which Fourier correlated data was determined largely by his view that Natural phenomena behave in a homogeneous way, and are subject to simple Laws. In the first sentence of his treatise on heat transfer, he states:

Primary causes are unknown to us; but are subject to simple and constant laws, which may be discovered by observation . . .

Fourier did in fact discover the Laws by observation. He states:

I have deduced these laws from prolonged study and attentive comparison of the facts known up to this time; all these facts I have observed afresh in the course of several years with the most exact instruments that have hitherto been used. (p. 2 of Preliminary Discourse.)

Fourier was a world class mathematician, and he would have described the observed proportionality with an equation of the form of Eq (3):

$$q = 0.213\ \Delta T \qquad (3)$$

But Fourier would have had no use for an equation of the form of Eq (3) for two reasons:

- Eq (3) is not the global Law which Fourier sought. It is a specific equation which applies only to the configuration tested, and only over the range tested.

- Eq (3) is not homogeneous--its terms are not dimensionally identical.

Therefore it was necessary for Fourier to solve the following problem:

How is it possible to describe, in a global and homogeneous way, the fact that the Table 1 data indicate q in kW/m^2 is numerically equal to 0.213 times ΔT in degrees C?

Fourier solved this global problem in a simple, imaginative way. He hypothesized that equations like Eq (3) are inhomogeneous because a parameter is missing from the equation, and the missing parameter has the dimensions which would make the equation homogeneous. Fourier christened the missing parameter "h", and arbitrarily assigned it the dimensions which brought homogeneity to the global form of Eq (3).

In the two excerpts below, Fourier first defines his brainchild "h", and then states that he achieved homogeneity by assigning dimensions to h and k:

We have taken as the measure of the external conducibility of a solid body a coefficient h, which denotes the quantity of heat which would pass, in a definite time (a minute), from the surface of this body, into atmospheric air, supposing that the surface had a definite extent (a square metre), that the constant temperature of the body was 1 (the temperature of boiling water at 76 cms mercury), and that of the air 0 (the temperature of melting ice), and that the heated surface was exposed to a current of air of a given invariable velocity. (Note that temperature is given in degrees Reamur.) (Article 36.)

In the analytical theory of heat, every equation expresses a necessary relation between the existing magnitudes x, t, v, c, h, k. . . If we attribute to each quantity its own exponent of dimension, the equation will be homogeneous, since every term will have the same total exponent (of dimension). (Article 161.)

Notice in the excerpt from Article 161 that Fourier *attributed* dimensions to h and k--ie he arbitrarily *selected* their dimensions--in order to *force* homogeneity in the global heat transfer equations. Also notice that Fourier violated the ancient Greek view of homogeneity in that he allowed dimensions to be multiplied/divided, as evidenced by his statement that *"every term will have the same (total) exponent (of dimension)"*.

Fourier presents a table which *"indicates the dimensions (attributed to) the three undetermined quantitites (x, t, v) and the three constants (c, h, k) with respect to each kind of unit"*. He then notes:

(If the dimensions in the table are applied) to the different (heat transfer) equations and their transformations, it will be found that they are homogeneous with respect to each kind of unit, . . . or some error must have been committed in the analysis . . .

If, for example, we take equation (b) of Art. 105, . . . we find that, with respect to the unit of length, the dimension of each of the three terms is 0; it is 1 for the unit of temperature, and -1 for the unit of time. (Article 162)

After Fourier arbitrarily assigned h and k the dimensions required for homogeneity, there was no longer any need for inhomogeneous equations such as Eq (3). They were replaced by homogeneous equations such as:

$$q = h \, \Delta T \tag{4}$$

$$q = k \, dT/dx \tag{5}$$

$$h = 0.213 \text{ kW/m}^2 \text{ C} \tag{6}$$

$$k = 1.73 \text{ kW/m C} \tag{7}$$

Fourier's brainchildren, h and k, did in fact bring dimensional homogeneity to heat transfer science.

In summary:

- **Fourier considered homogeneity axiomatic.**

- **Fourier achieved homogeneity by inventing h and k, and arbitrarily assigning them the dimensions required for homogeneity.**

CAUSE-AND-EFFECT PHENOMENA

Much of engineering science is concerned with cause-and-effect phenomena, and with equations which describe their behavior. In order to simplify our discussion, let us define "correlation" to mean "a parametric expression which describes a Natural cause-and-effect phenomenon". Qualitative correlations identify cause-and-effect pairs. Quantitative correlations identify cause-and-effect pairs, and describe functionality in a specific way or a general way. Examples of qualitative correlations are:

$$q = f\{\Delta T\} \tag{8}$$

$$W = f\{\Delta P\} \tag{9}$$

$$I = f\{V\} \tag{10}$$

$$\sigma = f\{\epsilon\} \tag{11}$$

Eq (8) states that heat flux q is an unspecified function of temperature difference ΔT, and similarly for Eqs (9) through (11). In each equation, the functionality is unspecified because the observed behavior varies widely, and cannot be described in a universal way by a particular function. For example, the function in Eq (10) is linear for Ohm's Law resistors, and highly nonlinear for transistors.

It is important to note that Eqs (8) through (11) contain the fundamental ideas of physical science, yet they play no part in modern engineering because they are not homogeneous.

A NEW VIEW OF DIMENSIONS AND DIMENSIONAL HOMOGENEITY

The modern view of dimensional homogeneity is inconsistent in that it permits dimensions and quantities of different dimension to be multiplied/divided, but does *NOT* permit them to be added/subtracted. The ancient Greek view is consistent in that no mathematical operation is permitted on dimensions or on quantities of different dimension, but the narrowness of this view greatly complicates descriptions of Natural behavior.

Now let us develop a new view of dimensions and dimensional homogeneity which is both consistent and broad. Let us begin by identifying numbers, dimensions, and dimensioned quantities:

- *Numbers* answer the question "How many?".

- *Dimensions* answer the question "Of what?".

- *Dimensioned quantities* answer the question "How many of what?".

Numbers may obviously be added/subtracted/multiplied/divided. Thus our task is reduced to deciding which mathematical operations may or may not be performed on dimensions and on dimensioned quantities.

In Fourier's view, and in the modern view, dimensions may not be added or subtracted, but they may be multiplied or divided. For example, meters may not be added to seconds, but meters may be divided by seconds to give m/s. The symbolism "m/s" means "meters divided by seconds".

But can meters really be divided by seconds? If meters could be divided by seconds, then it would be possible to answer the question:

How many seconds are in one meter?

Of course it is *NOT* possible to answer this question because a meter can *NOT* be subdivided into so many seconds. Meters and seconds have nothing to do with "How many?". They are dimensions, and dimensions have to do only with "Of what?". Similarly, addition/subtraction/multiplication/division have nothing to do with "Of what?"--they have to do only with "How many?".

It is axiomatic that:

- *Only* numbers can be added/subtracted.

- *Only* numbers can be multiplied/divided.

Mathematical operations can be performed only on numbers. Mathematical operations can *NOT* be performed on dimensions, in spite of the current view that dimensions *CAN* be multiplied/divided.

But if dimensions can not be multiplied/divided, how can dimensioned quantities be multiplied/divided? How can 100 meters be divided by 5 seconds if meters can not be divided by seconds? How can it be stated that velocity is 20 meters/second if it is not possible to divide meters by seconds?

In the new view of dimensional homogeneity, the answer to this seeming anomaly is:

When dimensioned quantities are multiplied or divided, only the numerical values are multiplied or divided.

In the new view, meters can NOT be divided by seconds, but the *NUMBER* of meters may be divided by the *NUMBER* of seconds to give the *NUMBER* of meters traversed in a second.

In the new view, the number of meters (100) is divided by the number of seconds (5), and the resultant velocity is 20 meters traversed in a second. If a symbolic way to write this result is desired, the symbolism should *NOT* indicate that meters is divided by seconds. If the symbolism now used to specify dimension is retained, it must be emphasized that the symbolism refers to mathematical operations performed only on the numerical values of dimensioned quantities, and *NOT* on the dimensions themselves.

In summary, the new view of dimensions and dimensioned quantities differs from the modern view in that:

- **Mathematical operations may *NOT* be performed on dimensions.**

- **When mathematical operations are performed on dimensioned quantities, the operations are performed *ONLY* on the numerical values of the dimensioned quantities.**

THE NEW VIEW OF THE EQUAL SIGN

In modern engineering science, the equal sign indicates dimensional identity as well as numerical equality, and therefore the equal sign indicates the homogeneity which is now deemed essential. But in the new view, correlations are generally inhomogeneous, and therefore it is necessary to have some symbol which indicates numerical equality, but makes no statement about dimensional identity.

A new symbol could be invented for this purpose, or the equal sign could be redefined. For the sake of discussion, let us assume that the equal sign is redefined to indicate numerical equality, but to make no statement about dimensional identity.

THE INTERPRETATION OF INHOMOGENEOUS EQUATIONS

It is common engineering practice to write homogeneous equations in terms of symbols which denote both parameter *and* dimension. For example, a Nomenclature section would indicate that the symbol "h" denotes the heat transfer coefficient in some specific dimension--perhaps kW/m^2C. In writing inhomogeneous equations, it is also advisable to have symbols which denote both parameter *and* dimension.

With symbols which denote both parameter *and* dimension, and with the equal sign redefined so that it indicates numerical equality but makes no statement about dimensional identity, equations are interpreted in the manner illustrated in the following example:

$$\Delta T = 1.78\ q/V^{.8} \qquad (12)$$

Eq (12) states "ΔT is numerically equal to 1.78 times the numerical value of q divided by the numerical value of V raised to the 0.8 power". (Note that dimensions are implicit in the symbols.)

It is important to note that:

- Each symbol denotes dimension as well as parameter, and therefore the appropriate dimensions are specified in the Nomenclature in the usual manner.

- The equal sign indicates numerical equality, but makes *NO STATEMENT* about dimensional identity.

- Mathematical operations are performed *ONLY* on the numerical values of the parameters.

- *No* significance is attached to homogeneity. Note that Eq (12) is not homogeneous.

THE DIFFICULTY WITH THE ANCIENT GREEK VIEW AND THE MODERN VIEW

The difficulty with the ancient Greek view is the conclusion that, because mathematical operations can not be performed on dimensions, they can not be performed on dimensioned quantities of unlike dimension. In the new view, mathematical operations can be performed on dimensioned quantities of unlike dimension because only their numerical values are operated on. Therefore there is no contradiction in the new view that mathematical operations *CAN* be performed on dimensioned quantities, but can *NOT* be performed on dimensions.

The difficulty with the modern view is the observation that dimensions can be multiplied or divided, when in fact dimensions can *NOT* be multiplied or divided, any more than they can be added or subtracted.

Recall Fourier's observation that his contemporaries did not know that conductive heat transfer depends on the length and area of the heat flow path, and therefore their equations were inhomogeneous, and therefore their equations were incorrect. But Fourier's logic is faulty. The equations were incorrect because they omitted important parameters--not because they were inhomogeneous. Correct equations require correct parameters and correct functionality, and these have nothing to do with homogeneity.

Fourier would agree with the statement by Langhaar (1951) that *"dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous"*. But that is in fact what Fourier did in a round about way. Recall that Fourier's methodology involved the following:

- Correlate convective heat transfer data and note that q is proportional to ΔT.

- Describe the observed proportionality with an equation which states that heat flux equals a proportionality constant times the ΔT across the boundary layer.

- Hypothesize that the proportionality constant is the numerical value of a Natural parameter.

- Call the Natural parameter "h".

- Hypothesize that rigorous equations are necessarily homogeneous.

- Arbitrarily assign to h (ie to the proportionality constant or *number*) those dimensions which result in homogeneity.

Note that this methodology is a way of assigning dimensions to *numbers* (the observed proportionality constants) without seeming to do so.

The homogeneity in Fourier's global equations was contrived. Homogeneity was not a Natural result--it was an assumption--an assumption validated by invention. Fourier assumed homogeneity, then he invented parameters to which he assigned whatever dimensions would provide the homogeneity he had assumed in the first place.

This closed loop methodology is the methodology generally employed to arrive at homogeneous equations intended to describe the behavior of cause-and-effect phenomena.

The reality is that cause-and-effect phenomena exhibit *IN*homogeneous behavior. Therefore inhomogeneous correlations rigorously/scientifically describe the behavior of cause-and-effect phenomena.

FOURIER'S EXPERIMENT REVISITED

Now let us correlate the data from an experiment like the one described by Fourier, but without assuming homogeneity. Our objective is to describe the data in Table 1 with an equation which makes the same statement made by the data.

The equation must identify the cause-and-effect pair, and must quantitatively describe their relationship. Since the experiment was specific--ie since it did not include a wide spectrum of geometries and fluids and temperatures and pressures--the resultant equation will also be specific.

Inspection of Table 1 indicates that the data states:

> **In the range investigated, the heat flux in kW/m^2 is numerically equal to 0.213 times the temperature difference in degrees C.**

If it is possible to write an equation which makes precisely this statement, and if the equation is inhomogeneous, then we must conclude there is no evidence of homogeneity in the Table 1 data.

Now let us note that the Nomenclature section defines the symbols q and T in the following way:

> **q is heat flux in kW/m^2.**
>
> **T is temperature in degrees C.**

With q and T defined in this way, the Table 1 data is symbolically described by Eq (13):

$$q = 0.213\ \Delta T \tag{13}$$

With the new view that the equal sign indicates numerical equality and makes no statement about dimensional identity, Eq (13) states:

> **In the range investigated, the heat flux in kW/m^2 is numerically equal to 0.213 times the temperature difference in degrees C.**

Notice that Eq (13) makes *precisely* the same statement made by the data in Table 1, and the equation is *IN*homogeneous. We therefore conclude that:

- The Table 1 data is quantitatively/rigorously/scientifically/completely described by an equation which is *IN*homogeneous.

- Since the Table 1 data are described by an equation which is inhomogeneous, it is not true that data generally exhibit homogeneous behavior.

- Since data do not generally exhibit homogeneous behavior, it is not true that Natural phenomena are scientifically described only by correlations which are homogeneous.

It should be noted that the data in Table 1 is quite general. Therefore the above data analysis demonstrates that convective heat transfer data *generally* exhibit inhomogeneous behavior, and are *generally* described in a scientific way by equations which are inhomogeneous.

ANOTHER EXAMPLE OF INHOMOGENEOUS BEHAVIOR IN NATURE

Eq (14) is the fundamental equation of motion. For many decades, it has been the starting point for rigorous and scientific motion analysis.

$$s = f(t) \tag{14}$$

Notice that Eq (14) is inhomogeneous, yet it is universally viewed as a rigorous and scientific description of Natural behavior.

THE MODERN VIEW OF THE FUNDAMENTAL EQUATIONS OF HEAT TRANSFER SCIENCE

In Fourier's view, the fundamental equations he conceived for convective and conductive heat transfer did not merely identify cause-and-effect pairs--they also described his empirical conclusion that convective and conductive heat transfer phenomena generally exhibit proportional behavior. In other words, Fourier intended h and k to be constant coefficients in Eqs (15) and (16), and he intended these fundamental equations to be phenomenological and global descriptions of heat transfer behavior.

$$q_{convective} = h\ \Delta T \tag{15}$$

$$q_{conductive} = k\ dT/dx \tag{16}$$

It has long been recognized that convective heat transfer phenomena do *NOT* generally exhibit proportional behavior. Some are moderately nonlinear (such as free convection), and some are highly nonlinear (such as boiling). Therefore Eq (15) is no longer the phenomenological and global description of convective heat transfer. It is now "the defining equation for h"--meaning that it defines h to be a shorthand way to write $q/\Delta T$. And h is no longer a constant coefficient in Eq (15)--it is a *variable* in Eq (15).

The modern view of conductive heat transfer is the same as that held by Fourier. In other words, Eq (16) is the phenomenological and global description of conductive heat transfer behavior, and k is a constant coefficient whose value depends only on material and temperature.

NEW FUNDAMENTAL EQUATIONS OF HEAT TRANSFER SCIENCE

With the new view that Natural phenomena are rigorously/scientifically described by equations which are inhomogeneous, let us generate new fundamental equations for convective and conductive heat transfer.

Qualitative correlations identify cause-and-effect pairs, but do not describe functionality. Since Eq (14) is a qualitative correlation which has been satisfactory for many decades, let us use this same form, and simply substitute cause-and-effect pairs for s and t. In this way we obtain Eqs (17) and (18), the qualitative/fundamental/global/inhomogeneous/rigorous/scientific/new equations for convective and conductive heat transfer.

$$q_{convective} = f(\Delta T) \tag{17}$$

$$q_{conductive} = f(dT/dx) \tag{18}$$

These new, inhomogeneous equations make the following statements:

Eq (17) states "Convective heat flux, in its specified dimension, is numerically equal to an unspecified function of the boundary layer temperature difference in its specified dimension."

Eq (18) states "Conductive heat flux, in its specified dimension, is numerically equal to an unspecified function of the temperature gradient in its specified dimension."

Now let us attempt to make these equations more specific.

Because convective heat transfer phenomena exhibit various forms of behavior from proportional to highly nonlinear, a global equation for convective heat transfer can be no more specific than Eq (17).

On the other hand, conductive heat transfer phenomena generally exhibit proportional behavior, and therefore it is possible to write a global equation for conductive heat transfer which is more specific than Eq (18). This more specific equation is

$$q_{conductive} = a\ (dT/dx) \tag{19}$$

where "a" is a dimensionless and constant coefficient whose value depends only on material and temperature. It is important to note that Eq (19) is *IN*homogeneous. We do *NOT* hypothesize that "a" is the numerical value of a Natural parameter, and then arbitrarily assign it the dimensions required to force homogeneity in the manner of Fourier. The behavior of cause-and-effect phenomena is inhomogeneous, and therefore equations which describe their behavior *should* be inhomogeneous.

Eq (19) could serve as the fundamental equation for a new science of conductive heat transfer. The new science would be quite satisfactory until one day it was discovered that some new material did *NOT* exhibit conductive heat transfer behavior which is proportional. At that point, it would be necessary to abandon Eq (19) in favor of Eq (18).

In order to avoid abandoning Eq (19) at some future date, it seems advisable to accept Eq (18) as the fundamental equation for conductive heat transfer, and to consider Eq (19) to be a specific form which applies in a global way *today*.

THE FUNDAMENTAL PARAMETERS OF HEAT TRANSFER SCIENCE

Fourier's heat transfer science is based on "Newton's Law of Cooling" and "Fourier's Law". The parameters in these homogeneous equations are:

- q heat flux

- ΔT boundary layer temperature difference

- dT/dx temperature gradient

- h a shorthand way to write $q/\Delta T$--"heat transfer coefficient"

- k a shorthand way to write $q/(dT/dx)$--"thermal conductivity"

Equations (17) and (18) are the fundamental equations of heat transfer science based on the new view of homogeneity presented here. The parameters in these new equations are the same as those in "Newton's Law of Cooling" and "Fourier's Law" *EXCEPT* for h and k. Because h and k are necessary only to achieve homogeneity, they are abandoned along with the requirement of homogeneity.

The heat transfer science founded on Eqs (17) and (18) is presented by Adiutori (1974, 1989).

CONCLUSIONS

- Cause-and-effect phenomena generally exhibit inhomogeneous behavior.

- Cause-and-effect phenomena are rigorously and scientifically described by correlations which are inhomogeneous.

- Heat transfer phenomena exhibit behavior which is inhomogeneous.

- Because heat transfer behavior is inhomogeneous, "Newton's Law of Cooling" and "Fourier's Law" should be replaced by the inhomogeneous equations:

$$q = f\{\Delta T\}$$

$$q = f\{dT/dx\}$$

- Because "h" and "k" are necessary only to achieve homogeneity, they are abandoned along with the requirement of homogeneity.

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