DRIFTING INSTABILITY IN TWO PHASE HEAT TRANSFER SYSTEMS

by

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ABSTRACT

A number of articles in the literature have reported the occurrence of "drifting instability" in two phase heat transfer systems. To date, this phenomenon has defied a quantitative or even a qualitative understanding. This lack of understanding has prevented the designer from enhancing those design features which minimize the occurrence of this generally undesirable phenomenon. It is the purpose of this article to supply this understanding which, in turn, leads to the realization that the designer is not helpless in the face of this phenomenon.

INTRODUCTION

Instability in action is usually a rather mystifying phenomenon in which the system in question seems to be partially in charge of its own fate and does not seem to respond as it should. It may oscillate or vibrate or bump, seemingly of its own volition, or it may simply "drift off" from its initial set point to another of its own choosing. This latter type of instability is the subject of this article.

Since, with our present state of knowledge, it would seem that all systems must satisfy the time and space dependent continuity, energy, and momentum equations, the analysis of instability has generally been performed by solving these equations in their most general forms. This frontal approach is usually carried out by nodalizing the system and then solving the myriad of resultant equations with the aid of either an electrical analogue or a digital computer program. This ambitious frontal attack suffers from a number of serious disadvantages, the most outstanding of which are:

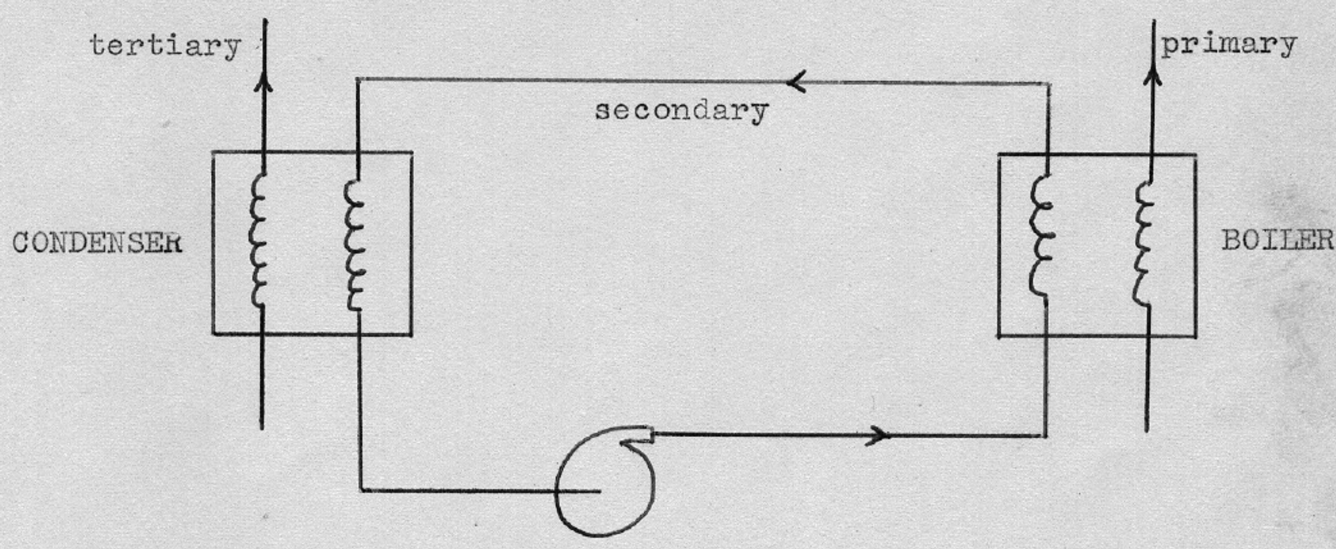
1. The solution, in order to require an analogue or computer of finite size, must usually be based on such a large number of assumptions of questionable accuracy that it is difficult to generate a high degree of confidence in the result.
2. Such an approach, by virtue of its inherent lack of understanding, does not lead to a clearcut answer as to how to optimize the design of a system to minimize the occurrence of the instability.
3. Such a solution generally requires so many equations that it is not possible to determine precisely where the culprit lies which is responsible for the instability.
4. Such an approach generally costs tens of thousands of dollars and requires man years of effort--while offering little hope of success.

It is the purpose of this paper to demonstrate that, through an understanding of thermal stability, we can arrive at a much better understanding of drifting instability and how it should be analyzed. By pursuing this understanding to its logical conclusion, we will arrive at a method of analysis which suffers from none of the above disadvantages. More important, this understanding leads to the realization that the designer can exert a great deal of control over the occurrence of the generally undesirable phenomenon known as "drifting instability".

SYSTEM DESCRIPTION AND ASSUMPTIONS

Before discussing the theory and analysis, it is helpful to describe the system to be analyzed and the assumptions on which the analysis will be based. The need for several of the assumptions may not be apparent at this point, but each is aimed at attaining the simplest solution possible without obscuring the real problem. Once we have focused on the crux of the problem, it then becomes an easy matter to remove the simplifying assumptions and thus increase the generality of the solution. (This article treats only the solution in its most simplified form. More general solutions are treated in a yet unpublished book by the author on the general subject of thermal stability.) The system to be treated in this article is shown in sketch (a):

Sketch (a)
System Schematic Diagram



The above system will be analyzed on the basis of the following simplifying assumptions:

1. The heat capacities of the primary and tertiary fluids

are infinite. The inlet temperatures can be set at desired levels and maintained invariant with time.

2. The pressure drop around the secondary system is negligibly small.

3. The overall heat transfer coefficient of the condenser is a constant which is independent of Q_c, T_s, and time.

4. The secondary system flow rate can be set and maintained at any desired value invariant with time.

5. The transport time around the secondary system is negligibly small.

It should be emphasized that the above assumptions are convenient for this discussion, but are not necessary. They are made only to simplify the solution to the point where the underlying cause of the drifting instability can be readily comprehended.

THEORY AND ANALYSIS

The theory of thermal stability (as it applies to drifting instability) centers about the following definition which seems adequate:

> A system such as that shown in sketch (a) is thermally stable, with respect to drifting, provided that, when $T_{s,\,iv}$ is perturbed to $(T_{s,\,iv} + \Delta T)$, equation (1) is satisfied:
>
> $$T_s\,(t = \infty) = T_{s,iv} \qquad (1)$$
>
> If the result of the perturbation does not satisfy equation (1), the system is thermally unstable with respect to drifting.

The analysis of drifting instability therefore centers about determining under what conditions equation (1) is or is not satisfied. This analysis is grossly simplified by the device of analyzing Q and its derivatives rather than employing the usual forms of the time and space dependent continuity, energy, and momentum equations. It will be seen that this simple device is almost singularly responsible for the simple stability criterion which results from the analysis. The analysis below is actually a sort of "dynamic heat balance" made on the secondary fluid. It is intended to determine under what conditions the dynamic heat balance will exert a restraint on thesecondary system temperature and thus prevent the temperature from drifting. In the absence of such restraint, the system will seem to drift of its own volition.

The analysis proceeds by noting that

$$Q_b(t) - Q_c(t) = Q_{stored}(t) \qquad (2)$$

and setting the boundary condition that

$$Q_{stored,iv}(t=0) = 0 \qquad (3)$$

The temperature of the secondary system is perturbed at t=0:

$$T_s(t=0+) = T_{s,iv} + \Delta T_s(t=0+) \qquad (4)$$

Therefore we can write

$$Q_b(t) = Q_{b,iv} + (dQ_b/dT_s)(\Delta T_s) \qquad (5)$$

$$Q_c(t) = Q_{c,iv} + (dQ_c/dT_s)(\Delta T_s) \qquad (6)$$

From (2), (3), (5), and (6)

$$Q_{stored}(t) = (dQ_b/dT_s - dQ_c/dT_s)(\Delta T_s) \qquad (7)$$

Now

$$Q_{stored}(t) = (dT_s/dt)(C_s) \qquad (8)$$

Therefore, from 7 and (8),

$$dT_s/dt = (\Delta T_s/C_s)(dQ_b/dT_s - dQ_c/dT_s) \qquad (9)$$

From equation (9), it can be seen that $T_{s,iv}$ is stable only if

$$(dQ_b/dT_s - dQ_c/dT_s) \leq 0 \qquad (10)$$

Equation (10) must be satisfied in order that $T_s(t)$ will tend toward $T_{s,iv}$, thus satisfying the definition of stability. This may be better visualized by noting that, if dT_s/dt is of the same sign as $\Delta T_s(t)$, then the latter will increase with time in an absolute sense--i. e. the perturbation will diverge from zero and the system will be unstable.

In many systems of practical importance, it is approximately true that

$$(dQ_c/dT_s)/(dQ_b/dT_s) \approx 0 \qquad (11)$$

in which case equation (10) can be further simplified to

$$(dQ_b/dT_s) \leq 0 \qquad (12)$$

Thus, the real culprit behind the drifting instability is the rather strange behavior peculiar to high quality boilers in which it is possible for dQ_b/dT_s to be greater than zero. If this derivative is greater than zero, it signifies that the heat transferred in the boiler will increase as a result of an increase in T_s. It should be noted that this differential behavior is just the opposite of what is normally encountered in a heat exchanger. In a differential sense, it is similar to heat flowing uphill--in violation of the second law of thermodynamics! This latter statement may be stretching the point, but it is interesting to note that this seeming violation of the second law results in the drifting instability. In a very real sense, the system is drifting in search of an operating condition wherein the heat can flow downhill in both an absolute and a differential sense.

Equation (10) shows that heat can be made to flow uphill in a differential sense by designing the condenser to have a large value of dQ_c/dT_s. This is one of the tools which the designer has at his disposal to eliminate or minimize the occurrence of drifting instability. This is discussed in greater detail in the Results section.

RESULTS

The major result of the above analysis is equation (10) which may be regarded as a stability criterion for the assumed and simplified system. This criterion would take different and more complicated forms if we were to remove some of the simplifying assumptions, but the underlying cause would of course remain the same.

In order to reach a better understanding of this criterion and how it applies to system design, it is instructive to rework the terms in equation (10) as follows:

$$Q_b = U_b^* (T_s)(T_p - T_s) \tag{13}$$

$$dQ_b/dT_s = -U_b^*(T_s) + (T_p - T_s)(dU_b^*/dT_s) \tag{14}$$

$$Q_c = U_c^*(T_s - T_t) \tag{15}$$

$$dQ_c/dT_s = U_c^* \tag{16}$$

Combining (10), (14), and (16) results in

$$(- U_b^* - U_c^* + (T_p - T_s)(dU_b^*/dT_s)) \leq 0 \tag{17}$$

Equation (17) indicates that the system stability can be improved by increasing the value of U_c^*. (This would of course require an increase in T_t in order to result in the required Q rating.) Though it is not obvious from equation (17), it is offerred without proof that the system stability could also be improved by increasing the thermal resistance of the boiler tube walls. (This would in turn require that the boiler surface area be increased in order to result in the desired value of U_b^*.)

It should also be noted that equation (17) will always be satisfied if the derivative term is zero. This derivative term represents the only non-linearity in the system and, as we might have expected a priori, the system is stable if it is completely linear.

CONCLUSIONS

The major conclusions which result from this article are:

1. The analysis of drifting instability is amenable to hand calculation in the manner described above.
2. The designer can exert a strong influence on the stability of the system with respect to drifting instability.
3. It is offerred without proof that the stability of many existing systems can also be improved by minor modifications to the auxiliary equipment and systems.

It should be emphasized that the purpose of this article is to promote an understanding of this drifting phenomenon rather than to analyze really practical systems. The system considered here is similar to what might be encountered in an experimental heat transfer facility, but the assumption of small pressure drop in the secondary system applies so poorly to power generation systems that theresults are not directly applicable. The analysis of a system in which this simplifying assumption is not made is covered in the aforementioned book and will be the subject of a later paper by the author.

SYMBOLS

C	system heat capacity, Btu/F.
Q	heat flow rate, Btu/hr
$\stackrel{s}{<}$	indicates that the system is stable if the inequality is satisfied
t	time, hr
T	temperature, F.
U^*	heat conductance, Btu/hr F.

SUBSCRIPTS

b	refers to	boiler
c	do	condenser
p	do	primary
s	do	secondary
t	do	tertiary
iv	do	initial value